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ATTITUDE POLARIZATION

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Abstract

Psychological evidence suggests that people's learning behavior is often prone to a "myside bias" or "irrational belief persistence" in contrast to learning behavior exclusively based on objective data. In the context of Bayesian learning such a bias may result in diverging posterior beliefs and attitude polarization even if agents receive identical information. Such patterns cannot be explained by the standard model of rational Bayesian learning that implies convergent beliefs. As our key contribution, we therefore develop formal models of Bayesian learning with psychological bias as alternatives to rational Bayesian learning. We derive conditions under which beliefs may diverge in the learning process and thus conform with the psychological evidence. Key to our approach is the assumption of ambiguous beliefs that are formalized as non-additive probability measures arising in Choquet expected utility theory. As a specific feature of our approach, our models of Bayesian learning with psychological bias reduce to rational Bayesian learning in the absence of ambiguity.

Keywords: Non-additive Probability Measures, Choquet Expected Utility Theory, Bayesian Learning, Bounded Rationality

JEL Classification Numbers: C79, D83

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1 Introduction

Several studies in the psychological literature demonstrate that people's learning behavior is prone to effects such as "myside bias" or "irrational belief persistence" (cf., e.g., Baron 2007, Chapter 9). For instance, in a famous experiment by Lord, Ross, and Lepper (1979), subjects supporting and opposing capital punishment were exposed to two purported studies, one confirming and one disconfirming their existing beliefs about the deterrent efficacy of the death penalty. Despite the fact that both groups received the same information, their learning behavior resulted in an increased "attitude polarization" in the sense that their respective posterior beliefs, either in favor or against the deterrent efficacy of death penalty, further diverged. Analogous results on diverging posterior beliefs in the face of identical information have earlier been reported by Pitz, Downing, and Reinhold (1967) and Pitz (1969) in the context of Bayesian updating of subjective probabilities. In violation of Bayes' update rule the subjects in these experiments formed biased posteriors that supported their original opinions rather than taking into account the evidence. The learning behavior elicited in these experiments cannot be explained by the standard model of rational Bayesian learning according to which differences in agents' prior beliefs must decrease rather than increase whenever the agents receive identical information. Models of rational Bayesian learning thus apparently ignore relevant aspects of real-life people's learning behavior.

In this paper we present formal models of Bayesian learning that allow for the possibility of a "myside bias". As our point of departure we assume that the paradigm of rational Bayesian learning may only be violated by agents who have *ambiguous* beliefs. That is, the beliefs of such agents cannot be described by additive probability measures alone but they additionally reflect the agent's personal attitudes. The impact of new information on an agent's beliefs is then two-fold. On the one hand, we take into account "rational" updating based on objective empirical evidence in accordance with the standard rational Bayesian learning hypothesis (cf., Tonks 1983; Viscusi and O'Connor 1984; Viscusi 1985). On the other hand, however, we also assume existence of a "myside bias" which results in an "irrational" enforcement of the agents' personal attitudes.

Our formal model is developed in two steps. In a first step we model ambiguous beliefs as non-additive probability measures, i.e., *capacities*, which arise in Choquet Expected Utility (CEU) theory (Schmeidler 1989; Gilboa 1987).[1] More specifically, we consider neo-additive capacities in the sense of Chateauneuf, Eichberger and Grant (2006) such that an agent's non-additive belief about the likelihood of an event is a weighted average

[1]CEU theory was originally developed to describe ambiguity attitudes that may explain Ellsberg paradoxes (Ellsberg 1961).

of an ambiguous part and an additive part. According to our interpretation, the additive part of the agent's belief is her best estimator for the "true" probability of a given event. The ambiguous part of her belief is relevant whenever the agent lacks absolute confidence in this estimator. This lack of confidence is resolved in our model by a parameter that measures the agent's optimistic versus pessimistic personal attitudes with respect to ambiguity.

In a second step we model the updating of ambiguous beliefs. According to our understanding of Bayesian learning, an agent with absolute confidence in her additive estimator should behave as a rational Bayesian learner. As a consequence, we assume that the additive part of the agent's posterior beliefs is governed by the standard rational Bayesian learning model of Viscusi (1985). In case there exists some ambiguity, we consider specific Bayesian update rules expressing different psychological attitudes towards the interpretation of new information (Gilboa and Schmeidler 1993). In particular, we analyze the consequences of the so-called *full Bayesian* (Pires 2002; Eichberger, Grant, and Kelsey 2006; Sinischalchi 2001, 2006) as well as the *optimistic* and the *pessimistic* update rules (Gilboa and Schmeidler 1993). An application of these update rules to some prior belief where the agent expresses ambiguity results in a Bayesian learning process that differs from rational Bayesian learning in that convergence to the "true" probabilities of some objective random process will - in general - not emerge. Rather, updating of beliefs reenforces optimistic, respectively pessimistic, attitudes of the agent thereby giving rise to learning behavior with a "myside bias".

Using this framework we then analyze the beliefs of two heterogeneous agents who have some prior beliefs, receive identical information and then update their beliefs according to some Bayesian update rule with psychological bias. Thereby, we differentiate between a *weak* and a *strong* form of myside bias. The weak form of myside bias is characterized by diverging posterior beliefs of the agents under repeated learning with identical information whereby the beliefs may move into the same direction. According to our interpretation the strong form of myside bias is equivalent to attitude polarization in that the posterior beliefs of the two agents move into opposite directions under repeated learning with identical information. To derive our main results we then consider two scenarios: In our first scenario the two agents have different initial beliefs and update their beliefs based on the same information by applying the same update rule. In our second scenario, the two agents receive the same information but apply different update rules. In both scenarios the resulting posterior beliefs may exhibit the weak as well as the strong form of myside bias. Notice that, in order to derive our result in the second scenario, we do not require that the agents have identical initial priors.

The remainder of our analysis is structured as follows. Section 2 presents the standard model of rational Bayesian learning of non-ambiguous beliefs and section 3 introduces ambiguous beliefs. Section 4 discusses updating of ambiguous beliefs under the three different update rules – full Bayesian, optimistic and pessimistic updating – that we consider in this paper. Section 5 then presents our main results on weak and strong myside bias in the form of diverging beliefs and attitude polarization. Finally, section 6 concludes.

2 Rational Bayesian learning

Consider the situation of an agent who is uncertain about the probability of an event, E, but can observe a statistical experiment with n independent trials where E is a possible outcome in each trial. Let

$$S = \times_{i=0}^{\infty} \{E, \neg E\}$$

denote the experiment's sample space, whereby $\neg E$ is the complement of E, and define

$$\begin{aligned} S_n &= \times_{i=0}^{n} \{E, \neg E\}, \\ S_{-n} &= \times_{i=n+1}^{\infty} \{E, \neg E\}. \end{aligned}$$

We can then formally describe the agent's information structure by the partitions

$$\mathcal{P}(n) = \{\{y\} \times S_{-n} \mid y \in S_n\}$$

where $n = 0, 1, ..., \infty$ denotes the n-th trial of the experiment. Denote by y^* the vector of outcomes observed by the agent. Since $\{y^*\} \times S_{-n} \in \mathcal{P}(n)$, after the n-th trial the agent knows the outcomes of the first n trials but not the outcomes of the remaining trials. For example, while the agent is totally ignorant with respect to the experiment's outcome before the first trial, i.e., $\mathcal{P}(0) = \{S\}$, she has perfect information after infinitely many trials, i.e., $\mathcal{P}(\infty) = \{\{y\} \mid y \in S\}$.

Suppose that the agent has a prior Beta probability distribution over the π parameter of a Binomial-distribution where $\pi(E)$ is the "true" probability of outcome E.[2] Further suppose that the agent resolves her uncertainty about π by an estimator $\tilde{\pi}(E)$ that is the expected value of this Beta-distribution, i.e., $\tilde{\pi}(E) = \frac{\alpha}{\alpha+\beta}$ for given distribution parameters $\alpha, \beta > 0$. More specifically, the prior Beta distribution has probability density

$$f(\pi) = \begin{cases} K_{\alpha,\beta} \pi^{\alpha-1} (1-\pi)^{\beta-1} & \text{for } 0 \leq \pi \leq 1 \\ 0 & \text{else} \end{cases}$$

[2] This Beta distribution model of rational Bayesian learning was introduced in the economic literature by Viscusi and O'Connor (1984) and Viscusi (1985).

where $K_{\alpha,\beta}$ is a normalizing constant.[3] Let $I_n \subset S$ denote the event that E has occurred k-times in n trials. Obviously, the information I_n is known to the agent after n trials since we have for the true outcome $\{y^*\} \times S_{-n} \subseteq I_n$. Further, denote by $f(\pi \mid I_n)$ the posterior probability density conditional on this sample information. Since the probability of receiving information I_n for a given π is, by the Binomial-assumption,

$$f(I_n \mid \pi) = \binom{n}{k} \pi^k (1-\pi)^{n-k},$$

we obtain by Bayes' rule

$$\begin{aligned} f(\pi \mid I_n) &= \frac{f(I_n \mid \pi) f(\pi)}{\tilde{\pi}(I_n)} \\ &= K_{\alpha+k,\beta+n-k} \pi^{\alpha+k-1} (1-\pi)^{\beta+n-k-1} \end{aligned}$$

whenever $\tilde{\pi}(I_n) = \int f(I_n \mid \pi) f(\pi) d\pi > 0$.

Observe that the agent's subjective posterior distribution over π is a Beta-distribution with expected parameters $\alpha + k, \beta + n - k$. Accordingly, the agent's posterior belief is given by the expected value of the posterior distribution, $\frac{\alpha+k}{\alpha+\beta+n}$, which, using that the prior belief is $\tilde{\pi}(E) = \frac{\alpha}{\alpha+\beta}$ and denoting the sample mean by $\mu_n = \frac{k}{n}$, we can rewrite as

$$\tilde{\pi}(E \mid I_n) = \left(\frac{\alpha+\beta}{\alpha+\beta+n}\right) \tilde{\pi}(E) + \left(\frac{n}{\alpha+\beta+n}\right) \mu_n. \tag{1}$$

That is, the agent's posterior is a weighted average of her prior and the sample mean whereby the weight attached to the sample mean increases in the number of trials.[4] Since, for every $c > 0$, $\lim_{n\to\infty} prob(|\mu_n - \pi(E)| \leq c) = 1$ we obtain the following result for this standard model of rational Bayesian learning.

Proposition 1: *Under the assumption of $\tilde{\pi}(I_n) > 0$ for all n the posterior belief $\tilde{\pi}(E \mid I_n)$ converges in probability to the true probability of event E if the number of trials, n, approaches infinity.*

As a consequence, the standard model of rational Bayesian learning cannot account for the learning behavior of agents whose posterior beliefs systematically diverge while they receive the same information.

[3] In particular, $K_{\alpha,\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$ where $\Gamma(y) = \int_0^\infty x^{y-1} e^{-x} dx$ for $y > 0$.

[4] Tonks (1983) introduces a similar model of rational Bayesian learning in which the agent has a normally distributed prior over the mean of some normal distribution and receives normally distributed information.

3 Ambiguous beliefs

We assume that individuals exhibit *ambiguity attitudes* in the sense of Schmeidler (1989) and who may thus, for example, commit paradoxes of the Ellsberg type (Ellsberg 1961). Following Schmeidler (1989) and Gilboa (1987), we describe such individuals as Choquet Expected Utility (CEU) decision makers, that is, they maximize expected utility with respect to *non-additive beliefs*. Properties of non-additive beliefs are used in the literature for formal definitions of, e.g., *ambiguity* and *uncertainty attitudes* (Schmeidler 1989; Epstein 1999; Ghirardato and Marinacchi 2002), *pessimism* and *optimism* (Eichberger and Kelsey 1999; Wakker 2001; Chateauneuf, Eichberger, and Grant 2006), as well as *sensitivity to changes in likelihood* (Wakker 2004). Our own approach focuses on non-additive beliefs that are defined as *neo-additive capacities* in the sense of Chateauneuf, Eichberger and Grant (2006).

Definition. *For a given measurable space* $(\Omega, \mathcal{F})$ *the neo-additive capacity,* ν*, is defined, for some* $\delta, \lambda \in [0,1]$ *by*

$$\nu(E) = \delta \cdot (\lambda \cdot \omega^o(E) + (1-\lambda) \cdot \omega^p(E)) + (1-\delta) \cdot \tilde{\pi}(E) \tag{2}$$

for all $E \in \mathcal{F}$ *such that* $\tilde{\pi}$ *is some additive probability measure and we have for the non-additive capacities* ω^o

$$\begin{aligned} \omega^o(E) &= 1 \text{ if } E \neq \emptyset \\ \omega^o(E) &= 0 \text{ if } E = \emptyset \end{aligned}$$

and ω^p *respectively*

$$\begin{aligned} \omega^p(E) &= 0 \text{ if } E \neq \Omega \\ \omega^p(E) &= 1 \text{ if } E = \Omega. \end{aligned}$$

Recall that a Savage-act f is a mapping from the state space Ω into the set of consequences X. For a finite state space the Choquet expected utility of Savage act f with respect to a neo-additive capacity ν is given as

$$CEU(f,\nu) = \delta \cdot \left(\lambda \cdot \max_{s\in\Omega} u(f(s)) + (1-\lambda) \cdot \min_{s\in\Omega} u(f(s))\right) + (1-\delta) \cdot \sum_{s\in\Omega} \tilde{\pi}(s) \cdot u(f(s)), \tag{3}$$

where $u : X \to \mathbb{R}$ is a von Neumann-Morgenstern utility function. Neo-additive capacities can be interpreted as non-additive beliefs that stand for deviations from additive beliefs such that a parameter δ (*degree of ambiguity*) measures the lack of confidence the decision maker has in some subjective additive probability distribution $\tilde{\pi}$. Obviously, if there is no ambiguity, i.e., $\delta = 0$, (3) reduces to the standard subjective expected utility representation of Savage (1954). In case there is some ambiguity, however, the second parameter λ measures how much weight the decision maker puts on the best possible outcome of alternative f when resolving her ambiguity. Conversely, $(1 - \lambda)$ is the weight she puts on the worst possible outcome of f. As a consequence, we interpret λ as an "optimism under ambiguity" parameter whereby $\lambda = 1$, resp. $\lambda = 0$, corresponds to extreme optimism, resp. extreme pessimism, with respect to resolving ambiguity in the decision maker's belief.

Finally, observe that for non-degenerate events, i.e., $E \notin \{\emptyset, \Omega\}$, the neo-additive capacity ν in (2), simplifies to

$$\nu(E) = \delta \cdot \lambda + (1 - \delta) \cdot \tilde{\pi}(E). \tag{4}$$

4 Updating ambiguous beliefs

In contrast to EU preferences, CEU preferences give rise to several possibilities for deriving ex post preferences, i.e., preferences conditional on the fact that some event has occurred, from ex ante preferences. In this section we focus attention on three perceivable Bayesian update rules for non-additive probability measures and apply them to neo-additive capacities. As such we discuss the so-called full (or generalized) Bayesian update rule (Eichberger, Grant, and Kelsey 2006), as well as the optimistic and the pessimistic update rules (Gilboa and Schmeidler 1993).

Define the Savage-act $f_I h : \Omega \to X$ such that

$$f_I h(s) = \begin{cases} f(s) & \text{for } s \in I \\ h(s) & \text{for } s \in \neg I \end{cases}$$

where I is some event. Recall that Savage's sure-thing principle claims that, for all acts f, g, h, h' and all events I,

$$f_I h \succeq g_I h \text{ implies } f_I h' \succeq g_I h'.$$

Let us interpret event I as new information received by the agent. The sure-thing principle then implies a straightforward way for deriving preferences $\succeq_I$, conditional on the new information I, from the agent's original preferences $\succeq$ over Savage-acts. Namely, we have

$$f \succeq_I g \text{ if and only if } f_I h \succeq g_I h \text{ for any } h, \tag{5}$$

implying for a subjective expected utility maximizer

$$f \succeq_I g \text{ if and only if } EU(f, \pi(\cdot \mid I)) \geq EU(g, \pi(\cdot \mid I))$$

whereby $\pi(\cdot \mid I)$ is the additive conditional probability measure derived from the Bayesian update rule, i.e., for all $E \in \mathcal{F}$,

$$\pi(E \mid I) = \frac{\pi(E \cap I)}{\pi(I)}.$$

In order to accommodate ambiguity attitudes as elicited in Ellsberg paradoxes, CEU theory drops the sure-thing principle. As a consequence, conditional CEU preferences are no longer derivable from (5) since the specification of the act h is now relevant (see Gilboa and Schmeidler 1993; Pires 2002; Eichberger, Grant and Kelsey 2006; Sinischalchi 2001, 2006 for a discussion of different Bayesian update rules).

Let us at first consider conditional CEU preferences satisfying, for all acts f, g,

$$f \succeq_I g \text{ if and only if } f_I h \succeq g_I h$$

where h is the so-called conditional certainty equivalent of g, i.e., h is the constant act such that $g \sim_I h$. The corresponding Bayesian update rule for the non-additive beliefs of a CEU decision maker is the so-called full Bayesian update rule which is given as follows (Eichberger, Grant, and Kelsey 2006)

$$\nu^{FB}(E \mid I) = \frac{\nu(E \cap I)}{\nu(E \cap I) + 1 - \nu(E \cup \neg I)} \tag{6}$$

where $\nu^{FB}(E \mid I)$ denotes the conditional capacity for event $E \in \mathcal{F}$ given information $I \in \mathcal{F}$.

Observation 2: *Let $E, I \notin \{\emptyset, \Omega\}$ and $E \cap I \neq \varnothing$. Then an application of the full Bayesian update rule (6) to a prior belief (4) results in the posterior belief*

$$\nu^{FB}(E \mid I) = \delta_I^{FB} \cdot \lambda + \left(1 - \delta_I^{FB}\right) \cdot \tilde{\pi}(E \mid I) \tag{7}$$

such that

$$\delta_I^{FB} = \frac{\delta}{\delta + (1-\delta) \cdot \tilde{\pi}(I)}. \tag{8}$$

Proof: If $E, I \notin \{\emptyset, \Omega\}$ and $E \cap I \neq \varnothing$, then

$$
\begin{aligned}
\nu^{FB}(E \mid I) &= \frac{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cap I)}{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cap I) + 1 - (\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cup \neg I))} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cap I)}{1 + (1-\delta) \cdot (\tilde{\pi}(E \cap I) - \tilde{\pi}(E \cup \neg I))} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cap I)}{1 + (1-\delta) \cdot (\tilde{\pi}(E \cap I) - \tilde{\pi}(E) - \tilde{\pi}(\neg I) + \tilde{\pi}(E \cap \neg I))} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cap I)}{1 + (1-\delta) \cdot (-\tilde{\pi}(\neg I))} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cap I)}{\delta + (1-\delta) \cdot \tilde{\pi}(I)} \\
&= \frac{\delta \cdot \lambda}{\delta + (1-\delta) \cdot \tilde{\pi}(I)} + \frac{(1-\delta) \cdot \tilde{\pi}(I)}{\delta + (1-\delta) \cdot \tilde{\pi}(I)} \tilde{\pi}(E \mid I) \\
&= \delta_I^{FB} \cdot \lambda + \left(1 - \delta_I^{FB}\right) \cdot \tilde{\pi}(E \mid I)
\end{aligned}
$$

with δ_I^{FB} given by (8).□

In addition to the full Bayesian update rule we also consider so-called *h-Bayesian update rules* for preferences $\succeq$ over Savage acts as introduced by Gilboa and Schmeidler (1993). That is, we consider some collection of conditional preference orderings, $\left\{\succeq_I^h\right\}$ for all events I, such that for all acts f, g

$$
f \succeq_I^h g \text{ if and only if } f_I h \succeq g_I h \tag{9}
$$

where

$$
h = (x^*, A; x_*, \neg A), \tag{10}
$$

with x^* denoting the best and x_* denoting the worst consequence possible and $A \in \mathcal{F}$. For the so-called *optimistic* update rule h is the constant act where $A = \emptyset$. That is, under the optimistic update rule the null-event, $\neg I$, becomes associated with the worst consequence possible. Gilboa and Schmeidler (1993) offer the following psychological motivation for this update rule:

> "[...] when comparing two actions given a certain event I, the decision maker implicitly assumes that had I not occurred, the worst possible outcome [...] would have resulted. In other words, the behavior given I [...] exhibits 'happiness' that I has occurred; the decisions are made as if we are always in 'the best of all possible worlds'."

As corresponding optimistic Bayesian update rule for conditional beliefs of CEU decision makers we obtain

$$\nu^{opt}(E \mid I) = \frac{\nu(E \cap I)}{\nu(I)}. \tag{11}$$

Observation 3: *Suppose $E, I \notin \{\varnothing, \Omega\}$. An application of the optimistic update rule (11) to a prior belief (4) results in the conditional belief*

$$\nu^{opt}(E \mid I) = \delta_I^{opt} + \left(1 - \delta_I^{opt}\right) \cdot \tilde{\pi}(E \mid I)$$

with

$$\delta_I^{opt} = \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(I)}.$$

Proof: Applying the optimistic Bayesian update rule to a neo-additive capacity gives, for $E \notin \{\varnothing, \Omega\}$,

$$\begin{aligned} \nu^{opt}(E \mid I) &= \frac{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cap I)}{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(I)} \\ &= \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(I)} + \frac{(1-\delta) \cdot \tilde{\pi}(I)}{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(I)} \cdot \tilde{\pi}(E \mid I) \\ &= \delta_I^{opt} + \left(1 - \delta_I^{opt}\right) \cdot \tilde{\pi}(E \mid I) \end{aligned}$$

such that

$$\delta_I^{opt} = \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(I)}.$$

□

For the *pessimistic* (or Dempster-Shafer) update rule h is the constant act where $A = \Omega$, associating with the null-event, $\neg I$, the best consequence possible. Gilboa and Schmeidler (1993):

> "[...] we consider a 'pessimistic' decision maker, whose choices reveal the hidden assumption that all the impossible worlds are the best conceivable ones."

The corresponding pessimistic Bayesian update rule for CEU decision makers is

$$\nu^{pess}(E \mid I) = \frac{\nu(E \cup \neg I) - \nu(\neg I)}{1 - \nu(\neg I)}. \tag{12}$$

Observation 4: *Suppose $E, I \notin \{\varnothing, \Omega\}$. An application of the pessimistic update rule (12) to a prior belief (4) results in the conditional belief*

$$\nu^{pess}(E \mid I) = (1 - \delta_I^{pess}) \cdot \tilde{\pi}(E \mid I)$$

with

$$\delta_I^{pess} = \frac{\delta \cdot (1-\lambda)}{\delta \cdot (1-\lambda) + (1-\delta) \cdot \tilde{\pi}(I)}.$$

Proof: Applying the pessimistic Bayesian update rule to a neo-additive capacity gives, for $E \notin \{\varnothing, \Omega\}$,

$$\begin{aligned}
\nu^{pess}(E \mid I) &= \frac{\nu(E \cup \neg I) - \nu(\neg I)}{1 - \nu(\neg I)} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \tilde{\pi}(E \cup \neg I) - \delta \cdot \lambda - (1-\delta) \cdot \tilde{\pi}(\neg I)}{1 - \delta \cdot \lambda - (1-\delta) \cdot \tilde{\pi}(\neg I)} \\
&= \frac{(1-\delta) \cdot \tilde{\pi}(E)}{1 - \delta \cdot \lambda - (1-\delta) \cdot (\tilde{\pi}(\neg I))} - \frac{(1-\delta)\,\tilde{\pi}(E \cap \neg I)}{1 - \delta \cdot \lambda - (1-\delta) \cdot (\tilde{\pi}(\neg I))} \\
&= \frac{(1-\delta) \cdot \tilde{\pi}(E)}{1 - \delta \cdot \lambda - (1-\delta) \cdot (\tilde{\pi}(\neg I))} - \frac{(1-\delta)\,\tilde{\pi}(\neg I)}{1 - \delta \cdot \lambda - (1-\delta) \cdot (\tilde{\pi}(\neg I))}\tilde{\pi}(E \mid \neg I) \\
&= \frac{(1-\delta) \cdot \tilde{\pi}(E)}{1 - \delta \cdot \lambda - (1-\delta) \cdot (\tilde{\pi}(\neg I))} \\
&\quad - \frac{(1-\delta)\,\tilde{\pi}(\neg I)}{1 - \delta \cdot \lambda - (1-\delta) \cdot (\tilde{\pi}(\neg I))}\left[\frac{\tilde{\pi}(E) - \tilde{\pi}(E \mid I) \cdot \tilde{\pi}(I)}{\tilde{\pi}(\neg I)}\right] \\
&= \frac{(1-\delta) \cdot \tilde{\pi}(I)}{\delta \cdot (1-\lambda) + (1-\delta) \cdot \tilde{\pi}(I)} \cdot \tilde{\pi}(E \mid I) \\
&= (1 - \delta_I^{pess}) \cdot \tilde{\pi}(E \mid I)
\end{aligned}$$

such that

$$\delta_I^{pess} = \frac{\delta \cdot (1-\lambda)}{\delta \cdot (1-\lambda) + (1-\delta) \cdot \tilde{\pi}(I)}.$$

□

5 Diverging posteriors and attitude polarization

In this section we derive our main results which formally link the updating of ambiguous beliefs to diverging posteriors and attitude polarization in Bayesian learning behavior. Consider two agents $i \in \{1, 2\}$ and let

$$\nu_i(E) = \delta_i \lambda_i + (1 - \delta_i) \cdot \tilde{\pi}_i(E)$$

denote the belief of agent i. If a posterior belief $\nu_i(E \mid I_n)$, $i \in \{1,2\}$, converges in probability to a unique limit, we simply write this limiting posterior belief as $\nu_i(E \mid I_\infty)$. That is, $\nu_i(E \mid I_\infty)$ satisfies for every $c > 0$

$$\lim_{n\to\infty} prob\left(\left|\nu_i(E \mid I_n) - \nu_i(E \mid I_\infty)\right| \leq c\right) = 1.$$

We assume that both agents receive the same information, i.e., $\mathcal{P}^1(n) = \mathcal{P}^2(n) = \mathcal{P}(n)$ for all n. Our formal definition of "attitude polarization" captures the idea that the agents' posteriors diverge rather than converge when their initial beliefs are different despite the fact that they receive the same information. Moreover, we allow for the possibility that the agents can observe arbitrarily many trials and consider posterior beliefs that obtain in the limit.

Our first assumption ensures that the standard model of Bayesian learning discussed in section 2 obtains as a special case whenever the beliefs are non-ambiguous, i.e., $\delta = 0$.

Assumption 1: *If agent $i \in \{1,2\}$ updates a neo-additive priors $\nu_i(E)$ conditional on information I_n, then the additive part of her posterior belief, i.e., $\tilde{\pi}_i(E \mid I_n)$, conforms with rational Bayesian learning (1) whereby $\tilde{\pi}_i(I_n) > 0$ for all n.*

By the following assumption, we restrict attention to the case in which differences in initial beliefs of agents can only be due to their respective optimism parameters λ^i, $i \in \{1,2\}$, under ambiguity.

Assumption 2: *Both agents have identical additive estimators, i.e., $\tilde{\pi}_1(E) = \tilde{\pi}_2(E) = \tilde{\pi}(E)$ for all $E \in \mathcal{F}$, as well as identical degrees of ambiguity, i.e., $\delta_1 = \delta_2 = \delta$.*

Since the information partitions $\mathcal{P}(n)$ become finer with increasing n, the corresponding sample information forms a nested sequence of events $I_0 \supset I_1 \supset ... \supset I_\infty$. The sequence of subjective probabilities $\tilde{\pi}(I_0), \tilde{\pi}(I_1), ...$ is therefore monotonically decreasing, implying the existence of a unique limit point

$$\lim_{n\to\infty} \tilde{\pi}(I_n) = \tilde{\pi}(I_\infty) \in [0,1].$$

Together with the above assumptions and proposition 1 this fact allows us to characterize the convergence behavior with respect to the different update rules discussed in section

4. By proposition 1, we have for the additive part of the beliefs convergence to the true probability of event E, i.e.,

$$\lim_{n\to\infty} prob\left(\left|\tilde{\pi}\left(E \mid I_n\right) - \pi\left(E\right)\right| \leq c\right) = 1$$

for every $c > 0$, so that we find for $i \in \{1,2\}$:

Lemma

(i) *Full Bayesian learning. For $E, I \notin \{\emptyset, \Omega\}$ and $E \cap I \neq \varnothing$,*

$$\nu_i^{FB}\left(E \mid I_\infty\right) = \delta_{I_\infty}^{FB} \cdot \lambda_i + \left(1 - \delta_{I_\infty}^{FB}\right) \cdot \pi\left(E\right)$$

with

$$\delta_{I_\infty}^{FB} = \frac{\delta}{\delta + (1-\delta) \cdot \tilde{\pi}\left(I_\infty\right)}$$

(ii) *Optimistic Bayesian learning. For $E, I \notin \{\emptyset, \Omega\}$,*

$$\nu_i^{opt}\left(E \mid I_\infty\right) = \delta_{I_\infty}^{opt} + \left(1 - \delta_{I_\infty}^{opt}\right) \cdot \pi\left(E\right)$$

with

$$\delta_{I_\infty}^{opt} = \frac{\delta \cdot \lambda_i}{\delta \cdot \lambda_i + (1-\delta) \cdot \tilde{\pi}\left(I_\infty\right)}$$

(iii) *Pessimistic Bayesian learning. For $E, I \notin \{\emptyset, \Omega\}$,*

$$\nu_i^{pess}\left(E \mid I_\infty\right) = \left(1 - \delta_{I_\infty}^{pess}\right) \cdot \pi\left(E\right)$$

with

$$\delta_{I_\infty}^{pess} = \frac{\delta \cdot (1-\lambda_i)}{\delta \cdot (1-\lambda_i) + (1-\delta) \cdot \tilde{\pi}\left(I_\infty\right)}$$

We are now ready to state and prove our main results. To focus our analysis we only consider interesting differences between the two heterogeneous agents. In particular, we differentiate between two relevant cases of heterogeneity. On the one hand, we consider full Bayesian learners who have different initial attitudes with respect to optimism under ambiguity implying different prior beliefs. On the other hand, we consider agents who may have identical prior beliefs but have different, i.e., optimistic resp. pessimistic, attitudes with respect to the interpretation of new information. As our first main result (proposition 2) we identify conditions under which posterior beliefs diverge such that the directed distance between the posterior beliefs of the two agents is strictly greater than the directed distance between their priors. That is, we consider *diverging posteriors* in the sense that

$$\nu_1(E \mid I_\infty) - \nu_2(E \mid I_\infty) > \nu_1(E) - \nu_2(E) \tag{13}$$

where $\nu_1(E) \geq \nu_2(E)$. For example, if there is an initial gap in the prior beliefs, the repeated learning of identical information will widen this gap whereby the posteriors may move in the same direction. We also refer to this divergence in beliefs as a *weak* form of myside bias.

Proposition 2. (Diverging Posteriors)

(i) *Assume that both agents are full Bayesian learners. Then inequality (13) is satisfied if and only if* $\delta \in (0,1)$, $\lambda_1 > \lambda_2$, *and* $\tilde{\pi}(I_\infty) < 1$.

(ii) *Assume that agent* 1 *is an optimistic whereas agent* 2 *is a pessimistic Bayesian learner. Then inequality (13) is satisfied if* $\delta \in (0,1)$, $\lambda_1 \geq \lambda_2$, *and* $\tilde{\pi}(I_\infty) < \lambda_1 \leq 1 - \lambda_2$.

Proof:

Part (i). Let $\lambda_1 > \lambda_2$ and observe that, by the lemma, (13), i.e.,

$$\begin{aligned} & \delta_{I_\infty} \cdot \lambda_1 + (1 - \delta_{I_\infty}) \cdot \pi(E) - (\delta_{I_\infty} \cdot \lambda_2 + (1 - \delta_{I_\infty}) \cdot \pi(E)) \\ > \ & \delta \cdot \lambda_1 + (1 - \delta) \cdot \tilde{\pi}(E) - (\delta \cdot \lambda_2 + (1 - \delta) \cdot \tilde{\pi}(E)), \end{aligned}$$

is equivalent to $\delta_{I_\infty} > \delta$, i.e.,

$$\frac{\delta}{\delta + (1 - \delta) \cdot \tilde{\pi}(I_\infty)} > \delta,$$

which holds if and only if $\delta \in (0,1)$ and $\tilde{\pi}(I_\infty) < 1$. Finally, observe that $\lambda_1 \leq \lambda_2$ violates (13).

Part (ii). By the lemma, (13) now becomes

$$\begin{aligned} & \delta_{I_\infty}^{opt} + \left(1 - \delta_{I_\infty}^{opt}\right) \cdot \pi(E) - \left(1 - \delta_{I_\infty}^{pess}\right) \cdot \pi(E) \\ > \ & \delta\lambda_1 + (1 - \delta) \cdot \tilde{\pi}(E) - (\delta\lambda_2 + (1 - \delta) \cdot \tilde{\pi}(E)), \end{aligned}$$

which is equivalent to

$$\delta_{I_\infty}^{opt} + \left(\delta_{I_\infty}^{pess} - \delta_{I_\infty}^{opt}\right) \cdot \pi(E) > \delta(\lambda_1 - \lambda_2).$$

This last inequality is obviously satisfied for all $\pi(E)$ and $\delta \in (0,1)$ if $\delta_{I_\infty}^{opt} > \delta$, i.e.,

$$\begin{aligned} \frac{\delta \cdot \lambda_1}{\delta \cdot \lambda_1 + (1-\delta) \cdot \tilde{\pi}(I_\infty)} &> \delta \Leftrightarrow \\ \lambda_1 &> \tilde{\pi}(I_\infty), \end{aligned}$$

and $\delta_{I_\infty}^{pess} \geq \delta_{I_\infty}^{opt}$, i.e.,

$$\begin{aligned} \frac{\delta \cdot (1-\lambda_2)}{\delta \cdot (1-\lambda_2) + (1-\delta) \cdot \tilde{\pi}(I_\infty)} &\geq \frac{\delta \cdot \lambda_1}{\delta \cdot \lambda_1 + (1-\delta) \cdot \tilde{\pi}(I_\infty)} \Leftrightarrow \\ 1-\lambda_2 &\geq \lambda_1. \end{aligned}$$

Finally, observe that $\lambda_1 < \lambda_2$ violates the assumption $\nu_1(E) \geq \nu_2(E)$.□

Our second main result (proposition 3) focuses on conditions that ensure attitude polarization. Attitude polarization in our sense is a stronger concept than mere divergence of posteriors in that it additionally requires that the posteriors move in opposite directions. More specifically, we consider *attitude polarization* such that

$$\nu_1(E \mid I_\infty) > \nu_1(E) \geq \nu_2(E) > \nu_2(E \mid I_\infty). \tag{14}$$

In order to further focus our analysis we thereby restrict attention to the case in which the subjective estimator coincides with the objective probability, i.e., $\tilde{\pi}(E) = \pi(E)$.

Proposition 3. (Attitude Polarization)

(i) *Assume that both agents are full Bayesian learners and let $\tilde{\pi}(E) = \pi(E) \in (0,1)$. Then inequality (14) is satisfied if and only if $\delta \in (0,1)$, $\lambda_1 > \lambda_2$, $\tilde{\pi}(I_\infty) < 1$, and*

$$\lambda_1 > \pi(E) > \lambda_2. \tag{15}$$

(ii) *Assume that agent 1 is an optimistic whereas agent 2 is a pessimistic Bayesian learner and let $\tilde{\pi}(E) = \pi(E) \in (0,1)$. Then inequality (14) is satisfied if $\delta \in (0,1)$, $\lambda_1 \geq \lambda_2$, and*

$$\tilde{\pi}(I_\infty) < \min\{\lambda_1, 1-\lambda_2\}. \tag{16}$$

Proof:

Part (i). For full Bayesian learners (14) becomes

$$\begin{aligned} & \delta_{I_\infty} \cdot \lambda_1 + (1-\delta_{I_\infty}) \cdot \pi(E) > \delta \cdot \lambda_1 + (1-\delta) \cdot \tilde{\pi}(E) \\ \geq \; & \delta \cdot \lambda_2 + (1-\delta) \cdot \tilde{\pi}(E) > \delta_{I_\infty} \cdot \lambda_2 + (1-\delta_{I_\infty}) \cdot \pi(E), \end{aligned}$$

which obviously implies $\lambda_1 > \lambda_2$ and thereby the middle inequality is strict. Under the assumption $\tilde{\pi}(E) = \pi(E)$, the first and the last inequality then hold if and only if $\delta_{I_\infty} > \delta$, i.e., $\delta \in (0,1)$ and $\tilde{\pi}(I_\infty) < 1$, compare proposition 2, part (i), as well as

$$\lambda_1 > \pi(E) > \lambda_2,$$

which proves the result.

Part (ii). Consider at first agent 1. Observe that

$$\begin{aligned} \nu_1(E \mid I_\infty) &> \nu_1(E) \Leftrightarrow \\ \delta_{I_\infty}^{opt} + \left(1 - \delta_{I_\infty}^{opt}\right) \cdot \pi(E) &> \delta + (1-\delta) \cdot \tilde{\pi}(E) \end{aligned}$$

which, under the assumption that $\tilde{\pi}(E) = \pi(E)$, is equivalent to $\delta_{I_\infty}^{opt} > \delta$, i.e.,

$$\frac{\delta \cdot \lambda_1}{\delta \cdot \lambda_1 + (1-\delta) \cdot \tilde{\pi}(I_\infty)} > \delta.$$

This proves that $\delta \in (0,1)$ and $\tilde{\pi}(I_\infty) < \lambda_1$ are necessary and sufficient conditions for $\nu_1(E \mid I_\infty) > \nu_1(E)$.

Consider now agent 2 and observe that

$$\begin{aligned} \nu_2(E) &> \nu_2(E \mid I_\infty) \Leftrightarrow \\ \delta + (1-\delta) \cdot \tilde{\pi}(E) &> \left(1 - \delta_{I_\infty}^{pess}\right) \cdot \pi(E) \Leftrightarrow \\ \delta + (1-\delta) \cdot \tilde{\pi}(E) &> \left(\frac{(1-\delta) \cdot \tilde{\pi}(I_\infty)}{\delta \cdot (1-\lambda_2) + (1-\delta) \cdot \tilde{\pi}(I_\infty)}\right) \cdot \pi(E) \end{aligned}$$

which, under the assumption that $\tilde{\pi}(E) = \pi(E)$, necessarily holds for any $\delta > 0$ if

$$\begin{aligned} \frac{\tilde{\pi}(I_\infty)}{\delta \cdot (1-\lambda_2) + (1-\delta) \cdot \tilde{\pi}(I_\infty)} &< 1 \Leftrightarrow \\ \lambda_2 &< 1 - \tilde{\pi}(I_\infty). \end{aligned}$$

This proves the result.□

Remark. While our results of propositions 2(i) and 3(i) are driven by the initial gap in prior beliefs, the results of propositions 2(ii) and 3(ii) build upon the different learning rules of the agents. According to condition (15) attitude polarization for full Bayesian learners rather occurs if the difference in initial beliefs is large, i.e., strong optimism of agent 1 versus strong pessimism of agent 2. Observe that, under the assumptions of proposition 3(ii), condition (16) ensures attitude polarization for an arbitrary ambiguity parameter $\delta \in (0,1)$ and arbitrary probability $\pi(E) \in (0,1)$. In contrast to the finding for full Bayesian learners, we may therefore encounter attitude polarization for optimistic and pessimistic Bayesian learners even in the case of identical prior beliefs if $\tilde{\pi}(I_\infty)$ is sufficiently small.

6 Conclusion

To account for phenomena such as "myside bias" or "irrational belief persistence" in people's learning behavior we propose formal models in which the interpretation of new information is prone to psychological bias. Based on a simplified representation of ambiguous beliefs we develop parsimonious representations of the agent's initial beliefs and updating processes. Thereby, we focus attention on three alternative update rules that are characterized by different degrees of optimism, respectively pessimism, in the interpretation of new information. As a specific feature to our approach, the resulting models of Bayesian learning with psychological attitudes reduce to the standard model of rational Bayesian learning in the absence of ambiguity. However, we show that a model with rational Bayesian learning alone results in convergent beliefs and is therefore not a suitable framework to account for phenomena such as a myside bias.

We then develop a two heterogeneous agents setting to derive divergent posterior beliefs and attitude polarization for the agents' learning processes under ambiguity. Attitude polarization is defined as a stronger condition than divergent beliefs in that the posterior beliefs of the two agents move into opposite directions. While we assume that the agents receive the same information, the agents may have different prior beliefs or apply different learning rules. Two main findings emerge:

1. We may observe divergent posterior beliefs and attitude polarization for agents who have identical attitudes with respect to the interpretation of new information but have different initial attitudes with respect to optimism, resp. pessimism, under ambiguity;

2. We may observe divergent posterior beliefs and attitude polarization in case the agents have identical initial attitudes with respect to optimism, resp. pessimism, under ambiguity but have different attitudes with respect to the interpretation of new information.

Our stylized Bayesian learning models thus formally accommodate two alternative scenarios of a "myside bias". In a first scenario, a "myside bias" in the learning process arises because of personal attitudes towards the resolution of ambiguity. In a second scenario, a "myside bias" corresponds to personal attitudes towards the interpretation of information.

In future research we aim to apply our approach to topics in information economics that are typically analyzed under the assumption of rational Bayesian learning such as fictitious play in strategic games (see, e.g., Fudenberg and Kreps 1993; Fudenberg and Levine 1995; Krishna and Sjostrom 1998) or no-trade results (see, e.g., Milgrom and

Stokey 1982; Morris 1994; Neeman 1996; Zimper 2007). Along the line of heterogeneous agent models that depart from the rational expectations or rational Bayesian learning paradigms, our approach may also have promising implications for asset pricing models (see, e.g., Cecchetti, Lam, and Mark 2000; Abel 2002; Ludwig and Zimper 2006) and theories of the wealth distribution (see, e.g., Ameriks, Caplin, and Leahy 2003).

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