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**Human Capital and the Demographic
Transition: Why Schooling
Became Optimal**

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Human Capital and the Demographic Transition: Why Schooling Became Optimal*

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Abstract

This paper develops a model with increasing adult life expectancy as the driving force of the economic and demographic transition. We show that if parents invest their own time into children's human capital, rising adult life expectancy unambiguously increases fertility. With children educated in schools and parents paying tuition fees, the reaction of fertility to changes in longevity is ambiguous. If productivity of adult human capital is sufficiently large and parent's valuation for additional children is sufficiently low, fertility will decrease. Without a schooling system, rising life expectancy therefore initially increases fertility. As during the development process life expectancy rises, a schooling system will be endogenously adopted and the relationship between fertility and longevity reversed. We argue therefore that it is important to account for the change in the nature of the costs of child education: from time costs to monetary costs.

JEL Classification Numbers: J11, I12, N30, I20, J24.

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1 Introduction and Motivation

The history of humanity was – until recently – characterized by dismal economic conditions: low income, low life expectancy, low investment into human capital and high fertility. Briefly summarized, “the life of man [used to be], solitary, poore, nasty, brutish, and short” (Hobbes (1651), p. 78). In modern Western economies we observe the opposite: high income, high life expectancy, highly educated individuals and low fertility. In this paper we develop a model which is able to rationalize the monotonic increase in human capital investment, the hump-shaped relationship between fertility and life expectancy and the endogenous appearance of a formal schooling system. We argue that it is important to account for the change in the nature child education costs: from time costs in an underdeveloped economy to monetary costs in a developed economy.

The driving force of the model is rising adult life expectancy. We use a simple life-cycle setup in which adults differ with respect to their productivity on the labor market and decide about consumption, investment into adult and child human capital and the number of children. They also chose whether they educate their children at home or whether they endow them with human capital by sending them to a school and paying tuition fees. If parents increase children’s human capital using their own time, rising adult life expectancy will unambiguously increase fertility. On the contrary, if children are educated in a formal schooling system, agents’ reaction to rising life expectancy is ambiguous. We show that if adult human capital is sufficiently productive and parents’ preferences for children are sufficiently concave, fertility decreases as parents’ life expectancy rises. Furthermore, parents deciding to send their children to a school have – for any life expectancy – fewer children and invest more in their human capital.

The decision which system to adopt depends on parents’ ability level. As life expectancy – and thus lifetime income – increases, also less productive agents will opt for the formal schooling system. Thus, rising adult life expectancy induces a *composition* effect (formal vs. informal schooling) and a *behavioral* effect (effect of life expectancy can increase or decrease fertility if children are educated in schools). For initially low life expectancy, the share of agents participating in the formal schooling system is low. Higher life expectancy thereby increases economy-wide fertility. With rising life expectancy during the development process, formal schooling becomes efficient for more and more people, generating a drop in aggregate

fertility and an increase in human capital investment.

The key contribution of this paper is to provide a novel explanation for the fertility transition and the endogenous appearance of a mass schooling system in an otherwise rather standard model. The explanation is based on an endogenous change in the nature of investment in child quality from time costs to monetary costs. We thus propose a theory why a formal schooling system emerged endogenously *without* a state intervention on a large scale. What we do *not* explain is why eventually schooling become free, i.e. why the society – via government and parliament – decided to first subsidize private schooling and then set up a public schooling system financed by taxes. We leave an extension of the model by endogenous political decisions (tax system, educational institutions, etc.) for further research.

This paper is not the first to provide a possible explanation for the economic and demographic change transition. Possible causes for declining fertility are declining child mortality rates (Kalemli-Ozcan (2002), Kalemli-Ozcan (2003), Tamura (2006)), natural selection favoring parents with a higher preference for child quality than quantity (Galor and Moav (2002)) and the narrowing of the gender wage gap making children more expensive (Galor and Weil (1996)). Further explanations are changing marriage institutions with a rising proportion of better educated women (Gould, Moav, and Simhon (2008)), structural change and an increasing share of people investing into human capital (Doepke (2004)) or the introduction of compulsory schooling (Sugimoto and Nakagawa (2010)). In Cervellati and Sunde (2005), Cervellati and Sunde (2007) and Soares (2005) rising adult life expectancy serves as the key explanatory variable for the observed economic development.¹ Particularly, higher life expectancy induces agents to invest more into human capital and decrease fertility. The driving mechanisms are the increasing opportunity costs of fertility as adult's human capital investment rises. Empirical evidence for the differential impact of life expectancy on population growth is provided by Cervellati and Sunde (2009). They show that before the demographic transition, improvements in life expectancy primarily increased fertility. Using historical time series, Clark (2005b) argues that fertility is not monotonically related to income or life expectancy. This hypothesis is supported by Lehr (2009) based on data from contemporary developing countries.

¹There is an enormous amount of papers not explicitly targeting to explain the demographic transition but provide explanations for various other aspects of the development process. Prominent papers are Galor and Weil (2000), Hansen and Prescott (2002), Jones (2001), Kremer (1993), Strulik (2004), Kögel and Prskawetz (2001), Ehrlich and Lui (1991), Ngai (2004), Lagerlöf (2003), Fernández-Villaverde (2001), Tamura (2002), Greenwood and Seshadri (2005), and Bar and Leukhina (2010). An excellent overview is provided by Galor (2005).

More recently, the literature started to deal with the question why schooling systems came into existence. Galor and Moav (2006) develop a model with physical and human capital as complementary factors of production. They argue that as the accumulation of physical capital was running into diminishing returns, capitalists lobbied – out of a profit maximizing rationale – for the introduction of compulsory schooling in order to increase their workers human capital. In Boucekkine, de la Croix, and Peeters (2007), the appearance of a public schooling system emerges from profit maximizing behavior of municipalities. As population density increased, more schools were constructed decreasing the distance (of each agent to the next school leading to higher attendance rates. Restuccia and Vandenbroucke (2010) explain cross-country differences of educational attainment and the catching-up process of developing countries with higher life expectancy and technological progress.²

Section 2 provides stylized facts and section 3 contains a detailed description of the model and the solution to the individual choice problem. The dynamic behavior of the economy with a discussion of the development path and an illustrative simulation exercise can be found in section 4 whereas section 5 provides empirical evidence in favor of the model. Section 6 concludes the paper. All proofs are relegated to the appendix.

2 Life Expectancy, Schooling and Fertility

After a stagnation of living standards over centuries, the beginning of the 19th century was the starting point of an unprecedented change in almost all aspects of economic and social life.³ Increasing life expectancy became a trend rather than an occasionally lucky event (Fig. 1a). Improvements in living conditions of daily life were initially reflected in higher fertility. Crude birth rates and net reproduction rates reached their historical peaks around 1820 and started to fall soon thereafter (Fig. 1c). At the same time, acquisition of formal human capital started to gain momentum for the first time in history. The earliest statistics indicate that in 1850 around 10% of the children of age 5-14 attended primary school and secondary school (10-19 years) did not enter official statistics before 1900 when the demographic transition was

²For instance, Acemoglu and Robinson (2000), Bertocchi and Spagat (2004) and Grossman and Kim (2003) argue that providing education to the masses decreases the potential for social conflict and civic disorder. According to these papers, the introduction of a free (and compulsory) education system was not necessarily an altruistic act but served rather the interests of the ruling class.

³In this paper we use data for England and Wales but the same pattern can be also observed in other countries around the same time with good data; one prominent and often studied example being Sweden.

already well under way. Human capital measured by the literacy, however, was considerably higher. In the early 19th century, around 30% of all brides and 60% of grooms signed their marriage contracts with their names instead of using an “X” (Fig. 1c).⁴ Note that the timing of the fertility reversal is closer to the introduction of primary schooling than to investment into human capital in general (literacy).

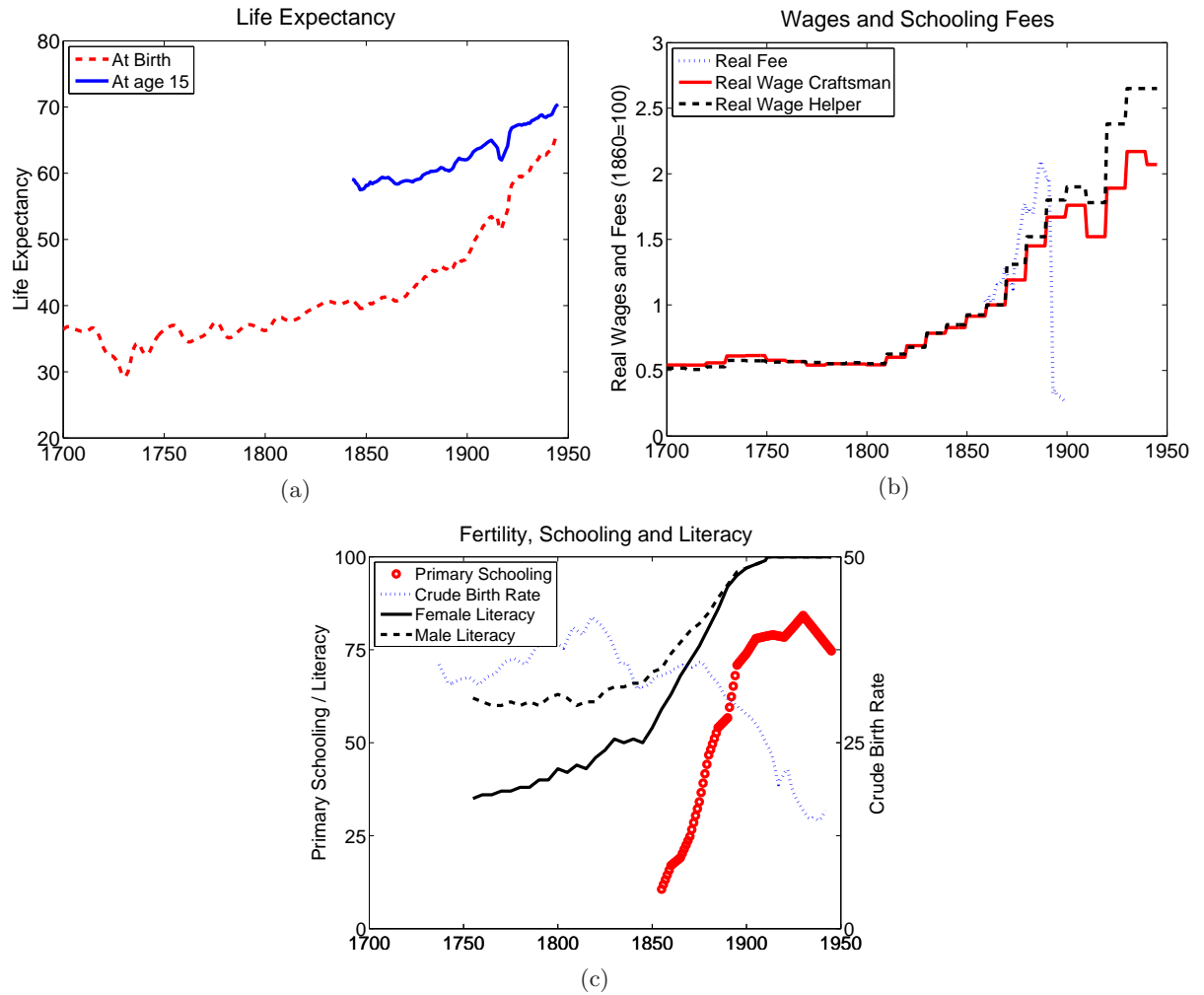
Looking at the history of British education reveals that primary schooling enrollment rates started to increase *before* the introduction of a compulsory schooling. The Elementary Education Act 1870 (also known as Forster’s Education Act) provided only partial funding for schools in underdeveloped regions but fees were still charged. The Elementary Education Act of 1880 made schooling compulsory for children aged 5-10 (but was never aggressively enforced) and only the Free Education Act of 1891 made basic education virtually free by heavily subsidizing primary schooling. Thus, education was initially not free but financed by parents. Data for 1834 Manchester show that up to 80% of children’s education was paid for only by parents (West (1970), p. 84) and approximately 1% of Net National Income in 1833 was spent on day-schools (West (1970), p. 87). The development of fees relative to wages is shown in figure 1b demonstrating that the rise of tuition fees kept pace with the general wage increase and even outpaced it shortly before the Free Education Act was enacted. This suggests that education became relatively more expensive with a *ceteris paribus* detrimental effect on educational investment. Nevertheless, we observe that some parents decided to send their children to costly schools. These families were most likely not member of the rich bourgeois (they could afford e.g. private tutors anyway) but belonged rather to the lower or middle class indicating that they recognized the value of education, were able and willing to pay for it.

3 The Model

In this section we describe the setup and solution to the model. The first subsection deals with the timing and notational conventions, followed by the description of aggregate production, production of human capital and the pricing of formal schooling. Then we move on to

⁴Whether literacy was a useful skill before and during the industrial revolution is hotly debated in the literature. After the seminal paper by Galor and Weil (2000) human capital has been accepted as *the* key ingredient of any unified growth theory. Mokyr (2004), however, claims that literacy was restricted to a small share of the population (government officials, military personnel or members of the aristocracy) and is unlikely to serve as a good explanatory variable.

Figure 1: Stylized Facts



Data sources: Crude Birth Rate: Chesnais (1992). Tuition Fees: Mitch (1986). School Enrolment Rates: Flora, Kraus, and Pfenning (1983). GDP per Capita: Maddison (2003). Literacy: Schofield (1973). Life Expectancy and Population Growth Rate: Wrigley and Schofield (1981) and Human Mortality Database (2008). Wages: Clark (2005a). The data refer to England and Wales. Enrollment data refer to the percentage of the age group (corresponding to the Net Enrollment Ratio).

the households' preferences and constraints and solve the individual maximization problem. Finally, we solve for the general equilibrium and discuss the dynamic behavior of the economy.

3.1 Timing and Conventions

Consider an overlapping generations economy in which adults live for $a_B + T$ years. T is the life expectancy of an adult agent who enters adulthood at age a_B which may be regarded as the “economic” birth.⁵ As a child, the agent may receive some education from her parents but is otherwise passive and does not make any own decisions. Adults can decide about consumption, number and education of children, and investment into adult human capital. Children are born right after parents enter adulthood at a_B . Parents can decide to educate children at home (“informal system”) or they can decide to send their children to school and pay tuition fees (“formal system”).⁶ Time investment into adult human capital increases productivity on the labor market and agents differ with respect to their labor productivity.

The economy is populated by a discrete number of overlapping generations and each new generation (cohort) is indexed by τ . The new household born at time t has a life expectancy of $a_B + T_t$ where the length of childhood a_B is time invariant whereas life expectancy will change during the development process. Life expectancy is identical across agents and determined by exogenous forces outside of the households' control. Investment into adult human capital, child human capital and the number of children are continuous variables.⁷ Reproduction is asexual, one agent in the present setting can be interpreted as a family making joint decisions. The notation in the paper is as follows: the subscripts τ and t denote cohort and calendar time, an individual's ability (type) is denoted by μ , a prime indicates a partial derivative, and the superscripts $j \in \{if, fo\}$ refer to the informal and formal schooling system. Variables with a bar (e.g. \bar{x}) denote averages and a tilde (e.g. \tilde{x}) indicates some threshold value for a variable. When no misunderstanding is expected, we omit indexes.

⁵Some papers, e.g. de la Croix and Licandro (2009) refer to this as puberty. In the context of this paper this could also be understood as marriage, see Voigtländer and Voth (2009) on the link of marriage and fertility.

⁶We use the terminology formal and informal to distinguish between education at home and education in some institution not requiring parents' time but money to “buy” the time of a teacher. In this model, formal education in a school is not free but parents have to pay tuition fees. See de la Croix and Doepke (2004) who model a framework with public (free but tax-financed) or private (tuition fees) school system and examine the long-run effects of the two educational system on growth and inequality.

⁷See Doepke (2005) for a model with discrete and sequential fertility decision. Although in presence of uncertainty the indivisibility assumption has an effect on the fertility behavior, he states that the “quantitative predictions of the models are remarkably similar”.

3.2 Aggregate Production

Human capital is the only productive factor in this economy and there is only one sector producing a homogeneous consumption good. We use a simple vintage model in which technological vintages are characterized by cohort specific productivity levels and each generation can operate only its cohort specific technology (each newborn generation automatically uses the new vintage).⁸ Thus, agents earn over their entire working life the output (wage) of that vintage. This allows us to concentrate only on the labor market equilibrium at one point in time for one generation and avoids making assumptions about the substitutability of agents over different ages and vintages of human capital. Aggregate production for a generation τ is given by the linear technology

$$Y_\tau = A_\tau \mathcal{H}_\tau, \quad (1)$$

where A_τ denotes cohort specific productivity and \mathcal{H}_τ is the aggregate stock of effective labor supply, respectively. Effective labor supply is defined as $\mathcal{H}_\tau = P_\tau \mathcal{L}_\tau$ where P_τ is the number of workers, and \mathcal{L}_τ is average effective labor supply per worker. To exhaust total production, wages per unit of human capital and per capita income are

$$\omega_\tau = A_\tau, \quad (2)$$

$$w_\tau = \omega_\tau \mathcal{L}_\tau. \quad (3)$$

Hence, wages per unit of human capital increase in the general level of productivity and income per capita increases with higher individual effective labor supply. As will become clear later, nothing hinges on the *absolute* level of income per capita or wages. In order to focus on the main predictions of the model, we abstain therefore from including a Malthusian element by introducing a concave production function with a fixed factor.

3.3 Human Capital of Adults

Upon becoming adults, agents may decide to spend h units of time on the acquisition of adult human capital. Agents' heterogeneity translates into different productivity on the labor

⁸See Cervellati and Sunde (2005) for similar assumptions. Chari and Hopenhayn (1991) develop a model showing that new technologies are not immediately adopted. In an empirical study Weinberg (2004) shows that older workers are more likely to operate old machines and new entrants into the labor market (young workers) will operate the most recent vintage.

market where ability μ is distributed uniformly on the $[0, 1]$ interval and shifts individual productivity linearly. We assume that adult human capital is produced according to

$$f(h) = \mu \frac{\bar{e}^\nu h^\theta}{\theta}, \quad (4)$$

with \bar{e}^ν denoting the positive effect of education adults received as children on their level of human capital.⁹

3.4 The Price of Education

Inputs into the production of education in the schooling system is only the time spent by teachers in the classroom. Further, we assume that working in a school makes agents relatively more productive teachers. Thus, the same agent educating children at home may be potentially less efficient in transmitting knowledge compared to the same person working in a school. We denote this “specialization-effect” by writing the teaching productivity of an agent with ability μ and human capital $f(h(\mu))$ as $m(f(h(\mu)))$ with $m(\cdot)$ being an increasing function. Schools hire teachers with different ability and pay a competitive hourly market wage ω per unit of human capital supplied. They maximize

$$\Pi = p \int_0^1 m(f(h(\mu))) \frac{q(\mu)^\epsilon}{\epsilon} d\mu - \omega \int_0^1 f(h(\mu)) q(\mu) d\mu, \quad (5)$$

where $q(\mu)$ is the number of hours worked by agents with ability μ and p is the market price of education.¹⁰ Demand of schools for each type is given by

$$q(\mu) = \left(\frac{p m(f(h(\mu)))}{\omega f(h(\mu))} \right)^{\frac{1}{1-\epsilon}}. \quad (6)$$

If teachers’ output is proportional to their level of human capital $m(\cdot) = f(h(\mu))$ and if agents’ teaching ability is not related to their level of human capital, we have $m(\cdot) = 1$. The choice of $m(\cdot)$ thus decides about the quality of the representative teacher. Total output of the

⁹The uniform distribution is not crucial for the main argument of the paper. Further, making child education more complementary to adult investment into human capital has only a quantitative effect but leaves the predictions of the paper unchanged.

¹⁰Since the production technology is concave schools will make profits. Returning these profits as lump-sum payments to the consumers would not change the qualitative results of the paper.

schooling system is then

$$\mathcal{E}_\tau^S = \frac{1}{\epsilon} \left(\frac{p}{\omega}\right)^{\frac{\epsilon}{1-\epsilon}} \int_0^1 \left(\frac{m(f(h(\mu)))}{f(h(\mu))}\right)^{\frac{\epsilon}{1-\epsilon}} m(f(h(\mu))) d\mu \quad (7)$$

where the market price of one unit of education time p is determined in equilibrium jointly with the demand side. Since the development of the price for schooling will turn out to be crucial for the development path, we will come back to this issue in section 4.2.

3.5 Household Preferences and Constraints

The agent's utility function follows rather standard assumptions. An agent likes consumption over the life-cycle and values educated children. Conditional on the decision $j \in \{if, fo\}$ how to educate children the agent from cohort τ maximizes the utility function

$$U^j = \int_0^T e^{-\rho a} \log c(a)^j da + \beta u(n^j z(e^j)) \quad (8)$$

where $\log c$ is the period utility obtained from consumption at age a , ρ is discounting future utility and $u(nz(e))$ is the intrinsic value of the quality-quantity composite weighted by β . The utility function u takes the number n of children times their quality which is captured by their human capital $z(e)$. This is a common assumption in the literature and can be understood as pure parental altruism or an implicit old-age pension system.¹¹ Human capital of children increases with investment e . The input into the production function is either parental time or teachers' time bought on the market for the price p per unit of time (tuition fees). Children survive with probability one until adulthood.¹² The timing of the agents' decisions is kept as simple as possible: agents first complete their investment into human capital and fertility and start working afterwards. Working on the labor market is an absorbing state and there is no retirement.

Since this paper does not focus on life-cycle dynamics or precise life-cycle profiles but rather on the trade-off between human capital investment (child and adult) and fertility, we assume

¹¹The alternative formulation in which agents derive utility from the utility of their children (i.e. the dynastic approach in the spirit of Becker and Barro (1988) or Barro and Becker (1989)) requires that agents know (or form expectations) what their children will do. For models where the old-age security motive is made explicit see e.g. Boldrin and Jones (2002) and Ehrlich and Lui (1991).

¹²We ignore uncertainty about child survival. See e.g. Kalemli-Ozcan (2003) and Strulik (2004) on the theoretical and Eckstein, Mira, and Wolpin (1999) or Ram and Schultz (1979) empirical relationship between child survival and investment into human capital.

that the discount rate and the interest rate are both zero.¹³ This simplifying assumption obviously eliminates the traditional life-cycle elements for consumption, saving and labor supply. However, this does not change the qualitative results and can be neglected since we do not make any quantitative statements. Using the assumptions from above, the problem can now be written as

$$\{j, c^j, h^j, n^j, e^j\} = \arg \max \quad T \log c^j + \beta u(n^j z(e^j)) \quad (9)$$

subject to the constraints

$$Tc^j \leq \omega f(h^j)(T - \phi n^j - h^j) - pe^j n^j, \quad \text{if } j = fo. \quad (10)$$

in the formal system where the education costs are monetary costs. In the informal system the constraint is¹⁴

$$Tc^j \leq \omega f(h^j)(T - (e^j + \phi)n^j - h^j), \quad \text{if } j = if. \quad (11)$$

In both constraints fixed time costs per child are denoted by ϕ . The problem is in general analytically not tractable and we make the following assumptions about functional forms

Assumption 1.

$$u(n, e) = \frac{(nz(e))^{1-\sigma}}{1-\sigma} \quad (12)$$

$$z(e) = \frac{e^\gamma}{\gamma} \quad (13)$$

with $\gamma < 1$, $\sigma > 0$.¹⁵

¹³Since marginal utility from consumption and the quantity-quality composite are independent, proceeding with $\rho = r \neq 0$ would not change the results (only the slope of the consumption profile would change), see the identical assumptions in e.g. de la Croix and Licandro (2009), Soares (2005) or Cervellati and Sunde (2005).

¹⁴In this setup we ignore the important issue of child labor. See e.g. Basu and Van (1998), Hazan and Berdugo (2002) or Baland and Robinson (2000) for models incorporating a child labor decision into growth models. However, note that we can rewrite the budget constraint by assuming that children can earn $n\varphi\omega f(h)$ where φ represents the relative wage of child labor. Then, $p = p^g - \omega f(h)$ and $\phi = \phi^g(1 - \varphi)$ where ϕ^g and p^g are *gross* and p and ϕ are *net* costs of schooling.

¹⁵We use the log-function for sub-utility from consumption to keep fertility independent from the *level* of ω – for a given T – as the economy is growing. A non-neutral effect of wages on fertility can be brought back into the model by choosing a utility function which does not balance income and substitution effects. See Jones, Schoonbroodt, and Tertilt (2008) for an excellent literature overview how in theoretical models income is related to fertility.

3.6 Individual Maximization Problem

The strategy is to solve the individual maximization problem – given wages and the price of schooling – conditional on the choice of the schooling system. This should highlight the conditional dynamics of fertility, investment into child, and adult human capital as life expectancy increases. Then we will analyze the effect of rising life expectancy on the choice of the parents' utility maximizing educational system. Using this two-stage procedure we can isolate the effect of rising life expectancy on the composition of the economy, i.e. formal vs. informal schooling and then the change in individual behavior conditional on this choices. Finally, we will put the individuals into a general equilibrium framework to allow for feedback effects and examine the dynamics of the aggregate economy. That is, we examine the simultaneous interaction of the behavioral and compositional change.

3.6.1 Household Solution in the Informal Education System

In this subsection we solve the household's problem assuming that agents educate their children in the informal system. At this stage, we do not ask which schooling system is optimal for parents but examine their behavior given that they decided to stay in the informal system. Thus, children's education consumes a share of their parents' time which is not available for productive work. Using λ to denote the multiplier attached to the resource constraint, the first order conditions¹⁶ of the problem (skipping the index $j = if$) are

$$\beta u'z - \lambda \omega f(h)(e + \phi) = 0 \quad (14)$$

$$\beta u'z' - \lambda \omega f(h) = 0 \quad (15)$$

$$f'(h)(T - (e + \phi)n - h) - f(h) = 0 \quad (16)$$

where a prime denotes partial derivatives. Due to perfect capital markets human capital investment h maximizes lifetime income and is independent of the marginal utility of consumption λ . Furthermore, combining the FOC's for fertility and child schooling we obtain

$$e = \phi \frac{\gamma}{1 - \gamma} \quad (17)$$

¹⁶To economize on notation we will not spell out the solution using the specified functional forms but rather use the general notation. For the proofs in the appendix we will, of course, switch to the specific functional forms whenever necessary.

implying that optimal investment into children’s human capital is independent of adult life expectancy, parents’ human capital (and ability) or wages. It is a constant determined by the relative time cost of children ϕ , and the properties of the human capital production function $z(e)$. Solving the entire household problem with informal education leads to the following proposition:

Proposition 1. *Assume that children are educated in the informal system. If adult life expectancy increases, adult schooling and fertility will increase. Investment into child human capital is constant.*

Proof. See Appendix. □

The intuition behind this result is rather simple. Adult schooling is rising since the time over which the benefits of educational investment can be reaped is increasing.¹⁷ The result is based on the trade-off between the opportunity costs of schooling today and future benefits. Child human capital is constant since the assumption made on u implies that the agent maximizes quasi-linear utility. Under this assumption marginal utility from child schooling and marginal costs are proportional in n . However, the price of one additional unit of fertility or child schooling is linearly increasing in $f(h)$ whereas returns to child schooling are concave. Thus, investment into child human capital does not change and income effects are absorbed by (rising) fertility. Fertility, on the other hand, is rising because of the intratemporal optimality condition between consumption and fertility. Rising life expectancy implies rising total lifetime income and the agent will – by concavity of both utility functions – distribute some of the additional “free” resources to increase consumption and fertility.¹⁸

3.6.2 Household Solution in the Formal Education System

Parents could also have their children educated by teachers ($j = fo$) for tuition fees p per unit of time. The crucial difference to the informal system is that the opportunity costs of child human capital are not valued by forgone adult wages but purely by monetary costs. Formally,

¹⁷This is the Ben-Porath (1967) mechanism. See however Hazan (2009) and Kalemli-Ozcan and Weil (2002) for opposite views on the link between life expectancy, human capital investment and lifetime labor supply (including the timing of retirement).

¹⁸This result can also be established by assuming only concavity of the production and utility functions and does not hinge on any functional form.

the first order conditions associated with the problem are

$$\beta u'z - \lambda(ep + \omega f(h)\phi) = 0 \quad (18)$$

$$\beta u'z' - \lambda p = 0 \quad (19)$$

$$f'(h)(T - h - \phi n) - f(h) = 0 \quad (20)$$

In contrast to the setup with informal education, child schooling costs are no time costs any more. Thus, they do not enter the equation determining the optimal solution for adult human capital but are only monetary costs valued by the marginal utility of consumption. Combining again the first order conditions for quantity and quality of children we obtain

$$e = \omega f(h) \frac{\phi}{p} \frac{\gamma}{1 - \gamma} \quad (21)$$

where now adult human capital increases also investment in child quality. This is an income effect stemming from the fact that the scarce factor time competing for labor supply and adult human capital accumulation is freed up. Thus, for the individual the “production function” for child human capital becomes linear instead of the convex costs caused by concave utility and concave production of adult human capital. However, the fact that child schooling is now purely a monetary cost implies also that the price of fertility relative to the price of schooling is rising in adult human capital (see FOC). Thus, it is not straightforward any more how fertility is changing with life expectancy (and h). Note that the introduction of a free public schooling system is still compatible with this setup. It is reasonable to assume that even without tuition fees the costs of schooling are bounded away from zero. This ensures that the household has a well defined demand for child education. Solving the household problem now allows us to state the following proposition

Proposition 2. *Assume that children are educated in the formal system. If adult life expectancy increases, adult schooling and child schooling will increase. Fertility is always rising for $\sigma \leq 1$ but may fall for $\sigma > 1$.*

Proof. See Appendix. □

Again, investment into adult human capital rises because of the horizon effect. Child human capital rises because the price of schooling does not increase in $f(h)$ and therefore

only the positive income effect is left over. Whether fertility increases or decreases depends on the coefficient of relative risk aversion with respect to the quantity-quality composite. Intuitively, if $\sigma \leq 1$, then the income effect of higher adult human capital will dominate (i.e. marginal utility from the quality-quantity composite is “less convex”). However, choosing $\sigma > 1$ allows the higher relative price of education to dominate (substitution effect). The reaction of fertility with respect to changes in life expectancy then depends mainly on the properties of the adult human capital production function. The more adult human capital increases, the more expensive quantity becomes relative to quality and therefore fertility is likely to decline.¹⁹

We can draw two main conclusions from analyzing household behavior under the two schooling regimes. Firstly, if there is no formal schooling system and the only input into children’s human capital is parental time we will not observe a decrease in fertility if adult life expectancy is increasing. Secondly, if there is a formal schooling system, a decrease in fertility is more likely but will not necessarily happen and depends on parameters of the utility and production function. If $\sigma > 1$, then fertility may decrease if $f(h)$ is increasing sufficiently as a consequence of rising adult human capital. The effect on aggregate fertility depends on the composition of the economy and on the parameters of production and utility function. However, to be able to match the observed stylized facts, we proceed for the rest of the paper with

Assumption 2.

$$\sigma > 1 \tag{22}$$

While interpreting the results, we have to keep in mind that we have operated in a highly stylized environment without any frictions. Returns to education are not affected by technological progress, an assumption frequently made in the literature.²⁰ The choice of log-utility for consumption implies that income effects – by simply raising wages (and p proportionally) – does not affect households’ allocations. The purpose of these assumptions was to isolate the

¹⁹An alternative interpretation of the economic mechanism is to assume that adults cannot invest into their own human capital but that they are equipped only with the education received as a child. Then, human capital rises exogenously for each new cohort if parents invest more into their children. In such a model, the costs of educating children at home also rises exogenously from cohort to cohort eventually inducing people to switch to the formal system.

²⁰See e.g. Schultz (1964), Foster and Rosenzweig (1996) or Bartel and Sicherman (1998) on the link between technological progress and investment into human capital. The appendix contains an extension where we allow for a positive correlation between parents’ ability as workers and teachers.

effect of different time allocation schemes on individual decisions. The quantitative relevance of this mechanism relative to competing explanations is ultimately an empirical question.

3.7 The Choice of Formal vs. Informal Schooling

Agents' optimal choice includes the decision in which education system their children are educated. For their decision, parents take their ability μ , wages ω , and the price of education p as given. The decision which schooling system to chose depends only on the potential income of parents. If they decide to send their children to a school, they do this because income earned on the labor market outweighs the costs of tuition fees. This is the sorting mechanism the model relies upon. The determinant is the wage-price ratio (ω/p) relative to potential earnings determined by individual ability and life expectancy. The more able agents are and the higher their life expectancy is, the cheaper and more efficient is education for their children in a formal system. Note that by assuming that ability is bounded from above, there may be such a vector $\{p, \omega\}$ that even the most able agents decide not to participate in the formal schooling system.²¹ Then, we can state the following proposition:

Proposition 3. *If there is a vector $\{p, \omega\}$ such that agents are indifferent between the formal and informal system, agents with $\mu \geq \tilde{\mu}$ will decide to educate their children in the formal system whereas agents with $\mu < \tilde{\mu}$ will decide to educate their children at home. This threshold ability level $\tilde{\mu}$ is decreasing with rising life expectancy.*

Proof. See Appendix. □

Proposition 3 means that for a given relative price structure, also less able agents find it optimal to switch to the formal system as life expectancy is rising. The economic explanation is that lower ability is partly compensated by higher investment into human capital. In turn, rising life expectancy increases optimal investment into adult human capital thereby increasing the opportunity costs of educating children at home also for less able agents. And obviously, if for an agent with ability $\mu = \tilde{\mu}$ it was optimal to join the formal system given T , this will be optimal for higher life expectancy too: the price of schooling is constant and not increasing in h , thus the agent is always better off in the formal system. Rising life expectancy therefore implies that the indifferent agent becomes less able as T rises. However, whether

²¹The standardization with the upper bound of $\mu = 1$ does not matter. It is obvious that for any finite bounded ability, there is a sufficiently high price to deter even the most able agent from participating in the formal school system.

this happens on the aggregate level once we allow for feedback effects of rising adult human capital investment on the price of education is not clear (see next section). Further, switching from the informal to the formal education system means that households' investment into human capital and fertility will change discontinuously.

Lemma 1. *If agents decide to educate their children in the formal system, they will increase investment in both types of human capital and decrease fertility.*

Proof. See Appendix □

This change in the optimal education system causes a change in educational attainment and fertility due to a changing composition but does not necessarily involve a behavioral change. Families choosing the formal system still could increase the number of children as their life expectancy increases. Moreover, heterogeneity in ability is now also reflected in the heterogeneity of decisions.

Lemma 2. *If children are educated in the informal system, ability does not change the solution to the households' problem. If children are educated in the formal system, higher ability increases adult and child human capital investment and decreases fertility.*

Proof. See Appendix. □

The negative correlation between parental education (ability) and fertility is a well documented and widely accepted fact. Note that for this pattern to emerge we need that the price of children's education is partly decoupled from parents' own human capital. Without the adoption of a formal schooling system, agents' allocations are identical despite different ability levels. Higher ability introduces an effect which proportionally increases prices of fertility and both types of human capital. Thus, more able agents enjoy only higher lifetime utility (due to higher consumption) without changing their allocation of time. Heterogenous behavior as a consequence of heterogeneity of skills requires that higher ability "buys" more time. This is, however, only the case if the price of child schooling is not perfectly linked to parents human capital.²²

²²See Skirbekk (2008) for a summary of the literature. He also shows that individuals with high status (or wealth) had also higher fertility rates until the beginning of the industrial revolution and thus rising investment into formal human capital.

3.8 Aggregation

Aggregate effective human capital in goods' production for a given generation τ is given by

$$\mathcal{H}_\tau = P_\tau \left[\tilde{\mu}_\tau f(h_\tau^{if}) \ell_\tau^{if} + \int_{\tilde{\mu}_\tau}^1 f(h_\tau^{fo}(\mu)) \ell_\tau^{fo}(\mu) d\mu - \int_0^1 q(\mu) f(h(\mu)) d\mu \right] \quad (23)$$

where the first term measures effective labor supply of agents educating their children at home. The second term measures total labor supply of agents educating their children in the formal system and the last term is the labor supply of teachers not available for producing consumption goods. Total education time purchased on the market and the equilibrium price are defined by

$$\mathcal{E}_\tau^D = \frac{\gamma\phi}{1-\gamma} \frac{\omega}{p} \int_{\tilde{\mu}}^1 f(h(\mu)) n(\mu) d\mu \quad (24)$$

$$\mathcal{E}_\tau^D(p(T)) - \mathcal{E}_\tau^S(p(T)) = 0 \quad (25)$$

Using the assumption of uniformly distributed ability in the population, average human capital in the economy and fertility for any cohort τ are

$$\bar{f}_\tau(h) = \tilde{\mu}_\tau f(h_\tau^{if}) + \int_{\tilde{\mu}_\tau}^1 f(h_\tau^{fo}(\mu)) d\mu \quad (26)$$

$$\bar{n}_\tau = \tilde{\mu}_\tau n_\tau^{if} + \int_{\tilde{\mu}_\tau}^1 n_\tau^{fo}(\mu) d\mu. \quad (27)$$

The first term is human capital and fertility of agents educating their children at home and the second term denotes the corresponding value for the families participating in the formal system.

4 The Dynamic System

The development process is shaped by the interaction of individually optimal decisions and macroeconomic externalities. Having solved the households' problem with fixed prices and for a given life expectancy, we will trace out the dynamics of simultaneous changes in prices and life expectancy. First, we study how the driving force of the model, adult life expectancy, is linked to the agents' individual decisions and how it evolves over time. Then, we will analyze the adjustment process of tuition fees when life expectancy is endogenous. Finally, we look at

the dynamics of demographic variables.

4.1 Life Expectancy

Research has led to mainly two competing explanations why life expectancy has increased over the last centuries: improvements in nutrition and progress in medical knowledge. Whereas e.g. Fogel (1997) argues that the increases in the intake of calories is responsible for decreasing mortality, Cutler, Deaton, and Lleras-Muney (2006) object in their survey that it was mainly progress of medical knowledge.²³ Although it seems reasonable that life expectancy at some time will reach a biological upper bound, there are no signs that this will happen within the next generations. In this paper we are agnostic about the sources of rising life expectancy and model it as a macroeconomic externality. Particularly, we link the next cohort's life expectancy to the average human capital of the current cohort, implicitly assuming that parents' knowledge, health behavior, etc. determines the life expectancy of their children. We formalize this by writing

$$T_{\tau+1} = \Psi(\bar{f}(h(T_\tau))), \quad (28)$$

where Ψ is a strictly concave and non-decreasing function capturing the positive externality of average human capital on life expectancy. To escape from a trivial solution we make

Assumption 3.

$$T_{\tau+1} - T_\tau = \Delta(T_\tau) = \Psi(\bar{f}(h(T_\tau))) - T_\tau \geq 0 \quad \forall T_\tau > 0. \quad (29)$$

This is a nonlinear difference equation leading to an arbitrary high but finite life expectancy. By imposing the restriction that Ψ is non-decreasing and strictly concave we rule out possible non-monotonicities on the development path. We do this for the sake of clarity of the paper's argument: the implications of a less restrictive specification of Ψ are that we may end up with no or more than one steady-state without gaining additional insights.

²³Cutler, Deaton, and Lleras-Muney (2006) show that there was no health-income gradient before the Age of Enlightenment and assert that ideas like germ theory, boiling water or simply washing hands are independent of the level of income. Mokyr (1993) takes a stand between the two theories writing that "...knowledge is believed to respond to market signals, social and political pressures, changes in incentives, institutions, and so on." In other words, as people escape from a state of mere subsistence, they can afford to spend resources on the advancement of knowledge.

4.2 Schooling Choice in General Equilibrium

As life expectancy rises, the price of education may increase or decrease depending on the assumptions made about the education sector. However, rising tuition fees do not matter as long as they are outweighed by sufficiently large increases in life expectancy. Put it differently: for any increase in potential income, there is a corresponding surge in the price of education such that the indifferent agent is characterized by exactly the same ability level. If prices increase by less, also less able agents will decide to join the formal system and $\tilde{\mu}$ will decrease. If the price of education rises faster then some less able agents will withdraw their offspring from formal schools. That is, the threshold ability level will increase undoing the positive effect of a higher potential income on children's education.

Proposition 4. *If life expectancy rises, the share of agents participating in the formal schooling system may increase or decrease. Without a positive specialization effect, $\tilde{\mu}$ will monotonically increase and may hit the upper bound of ability for sufficiently large life expectancy.*

Proof. See Appendix. □

For the sake of building some intuition, keep for a moment prices and wages fixed and assume and that there is no specialization effect, i.e. agents of different abilities are equally productive ($m(\cdot) = 1$). Rising life expectancy has two opposing effects on supply and demand in the education market. On the one hand, increasing life expectancy and adult human capital will then increase demand for formal schooling. On the other hand, profit maximization leads to less production of education time since the costs of teachers increase with their level of human capital but they are not becoming more productive. To bring supply in line with demand, the price of schooling must rise, thereby decreasing demand and increasing supply. This is the very reason why we need some specialization effect of teachers. Without such an effect, fewer agents would be willing to participate in the formal schooling system.²⁴ However, note that this argument does not require falling prices. The specialization effect must be just strong enough to prevent prices from going up “to much”.

²⁴See e.g. Bagnoud (1999) for Switzerland, Birchenough (1914) for England and Wales and Becker, Cinnirella, and Woessmann (2009) for Prussia for evidence on the reluctance of people (especially peasants and poor workers) to send their children to school despite compulsory schooling.

4.3 Population Dynamics

The dynamics of average (aggregate) fertility is ambiguous and depends on the strengths of the different mechanisms at work. Simplifying equation (27) leads to

$$\bar{n}_\tau = \tilde{\mu}_\tau \bar{n}_\tau^{if} + (1 - \tilde{\mu}_\tau) \bar{n}_\tau^{fo}. \quad (30)$$

We know that a rising share of parents participating in the formal system decreases fertility via the composition effect. However, if the weight of these families is initially small, this effect is likely to be dominated by the fertility of agents choosing the informal system. Secondly, even fertility of agents participating in the formal system may rise for a while as T rises. While the composition effect is then still working towards lower aggregate fertility, fertility of each subgroup will increase as life expectancy rises. Only if the share of the formal schooling is high enough and these agents also have fewer children as T goes up, will aggregate fertility of a cohort decrease unambiguously. Total population P_t and cohort size P_τ evolve according to

$$P_{t+1} = P_t + N_t(a) |_{a=a_B} \int_0^1 n(T_{t-a_B}, \mu) d\mu - N_t(a) |_{a=T_m}, \quad (31)$$

$$P_{\tau+1} = P_\tau (\bar{n}_\tau - 1), \quad (32)$$

where $T_m = T(t - T_m)$ denotes the life expectancy of the oldest agent in $t - 1$ (who dies in t) born T_m years ago. $N_t(a)$ is the number of adults in t who either are of childbearing age ($a = a_B$) or die ($a = T_m$) this period.²⁵ The number of newborns per agent of childbearing age is determined by its life expectancy T_{t-a_B} and ability μ , and is denoted by $n(T_{t-a_B}, \mu)$. Population growth rate for any stationary life expectancy T is implicitly defined by

$$g_P = \frac{n(T)}{T} \left[\frac{(1 + g_P)^T - 1}{(1 + g_P)^T} \right] \quad (33)$$

The behavior of the population growth rate can be described by the following corollary.

Corollary 1. *If fertility is a hump-shaped function of adult life expectancy, the population growth rate is also hump-shaped.*

Proof. See Appendix. □

²⁵Alternatively, population can be also written as the integral over all living cohorts $P_t = \int_{t-T_m}^{t+a_B} N_t(a) da$.

Population dynamics is slightly more complicated if we start from a stationary population and let life expectancy increase. Then we have initially a positive effect on the population growth rate due to higher fertility and a (delayed) positive effect due to the fact that old agents are living longer. However, the second effect is only transitory and vanishes as life expectancy settles at a constant value. Whether in the new steady state population is growing or shrinking depends on the fertility associated with the steady-state life expectancy.

4.4 Technological Progress

As outlined in the introduction, technological progress occurs through the invention of better and more productive machines which can be operated only by the cohort entering the labor force at the time of the introduction of the new vintage. Following the literature, we assume that higher level of human capital facilitates invention of more productive technologies²⁶ and model this by assuming

$$\frac{A_\tau}{A_{\tau-1}} = g(\bar{f}_{\tau-1}(h)), \quad (34)$$

with g increasing and concave. After having defined the static solution to the households' problem and specified how aggregate variables change over time, we are ready to define the equilibrium development path of the economy.

Definition 1 (Equilibrium). *Given an initial population P_0 and initial life expectancy T_0 , an equilibrium consists of a sequence of aggregate variables $\{\mathcal{H}_\tau, Y_\tau, A_\tau, T_\tau\}$, prices $\{p_\tau, \omega_\tau\}$, and individual decision rules $\{c_\tau^j, n_\tau^j, h_\tau^j, e_\tau^j, j\}$, $j \in \{if, fo\}$, such that*

1. *households optimality conditions given by equations (14) and (18) subject to the constraints (10) or (64) are satisfied,*
2. *aggregate variables are given by (1), (23), (28), and (34), prices by (3) and (25), and*
3. *life expectancy, total population, and cohort size evolve according to (28), (31), and (32).*

4.5 An illustrative simulation

The goal of this paper is to demonstrate the qualitative change in the behavior of agents as they endogenously decide to invest into the human capital of their children via a formal schooling

²⁶See e.g. Lucas (1988) or the excellent literature reviews by Jones (2005) and Klenow and Rodríguez-Clare (2005).

system. In this subsection we provide therefore only an illustrative simulation without any ambition to exactly match historical time series. Since the model lacks many realistic features, it would require a lot of “twisting and tweaking” of model parameters and a very lenient attitude with respect to the choice of functional forms which is of limited use as far as further insights is concerned. Especially, the model is not able to explain the drop in tuition fees caused by the introduction of the Free Education Act from 1891 see (Fig. 1b). We therefore restrict ourselves to the choice of rather simple functional forms and have to keep in mind that at some point of the development process, a more or less exogenous drop in p took place. Our choices for Ψ and m are

$$T_{\tau+1} = \delta T_{\tau} + \bar{f}_{\tau}(h)^{\alpha}, \quad (35)$$

$$m = f_{\tau}(h)^{\kappa}. \quad (36)$$

The parameters of the simulated model can be found in table 1. Figure 2 shows the basic patterns of the development process. Initially, life expectancy is low and only the most able agents invest into human capital of their children via the formal system. As life expectancy increases, despite a rising relative price of education the ability threshold $\tilde{\mu}$ decreases and more and more agents switch to the formal schooling system. Note that life expectancy and aggregate fertility are still rising. At this stage, average fertility in both systems is still rising and the composition effect is not sufficient to bring aggregate fertility down. However, this relationship changes during the development process. Despite the rising fertility of agents in the informal system, aggregate fertility is falling: the economy is now dominated by agents choosing the formal system and their fertility is falling as life expectancy keeps rising. On top of the compositional effect now also the behavioral effect works towards lower fertility.

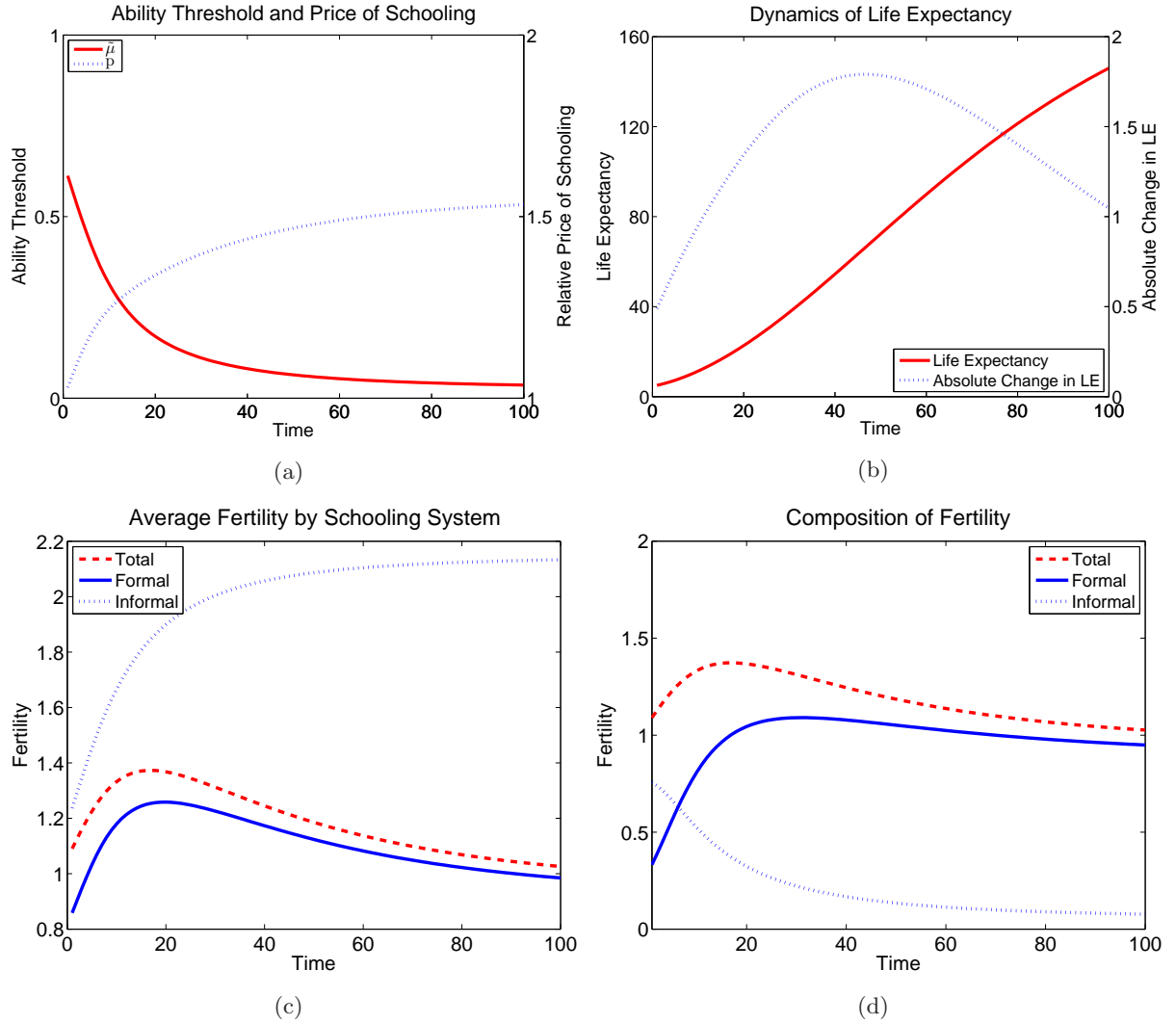
Table 1: Model Parameters for Simulation

β	γ	σ	η	ϕ	ρ	θ	κ	α	δ	τ	ϵ	ν
30	0.8	1.7	1.0	0.5	0.01	0.85	0.8	0.95	0.88	0.8	0.95	0.01

5 Empirical Evidence

The theory from above provides some testable predictions. Although we do not have access to detailed historical data, we can test our hypotheses using contemporaneous data and exploit

Figure 2: A Simulation Exercise



Data sources: Own simulations. See text and table for functional forms and parameter values.

the variation in time and between developing and developed countries. If the mechanisms at work are the same, we should be able to patterns in support of the above theory.

The main hypothesis to test is that the reaction of fertility to changes in adult life expectancy is non-monotonic and should depend on the presence of a primary schooling system (which in the model is a function of life expectancy). For low levels primary schooling enrollment rates (i.e. if most people choose $j = if$), rising adult life expectancy increases fertility whereas beyond some threshold level, higher adult life expectancy should have a negative total effect on fertility (since most agents choose $j = fo$). This correlation should also hold

if we additionally condition on adult human capital. Although higher adult human capital investment (holding primary schooling fixed) has a direct negative effect on fertility (via the composition effect) this should not change the basic pattern.

To test these hypotheses, we will estimate reduced form equations. The dependent variable (fertility) is the net replacement rate from the United Nations (2007). As a proxy for adult life expectancy we use the probability of a 15 year old agent to survive until the age of 60 in a given year from the World Development Indicators (The World Bank (2010)). To proxy for the availability of a primary schooling system, we use the share of 5-10 year old children with (primary) education from Barro and Lee (2010). This measure is more suitable than e.g. average years of schooling since it is naturally bounded between zero and unity and can be interpreted as the empirical counterpart to $\tilde{\mu}$. As a proxy for adult human capital we use years of secondary schooling in the population older than 15. To account for income effects (which we have explicitly ruled out in the theoretical model) we include GDP per capita from the Penn World Tables 6.3 (Heston, Summers, and Aten (2009)).

The data set covers 135 countries with up to four observations per country. Figure 3 shows a scatterplot with the adult survival rates and the net replacement rate.²⁷ Due to the restrictions imposed by the availability of data, the sampling period used for estimation starts in 1960 and ends in 2000 with observations in an interval of ten years. The estimated models suffer from strong endogeneity problems whereas the number of available instruments is limited and their applicability is highly debated in the literature. Therefore, we do not aim at establishing causality by using weak or invalid instruments but will rather show that the stylized facts presented in the historical time series and motivating the theoretical part are also present in contemporaneous data.

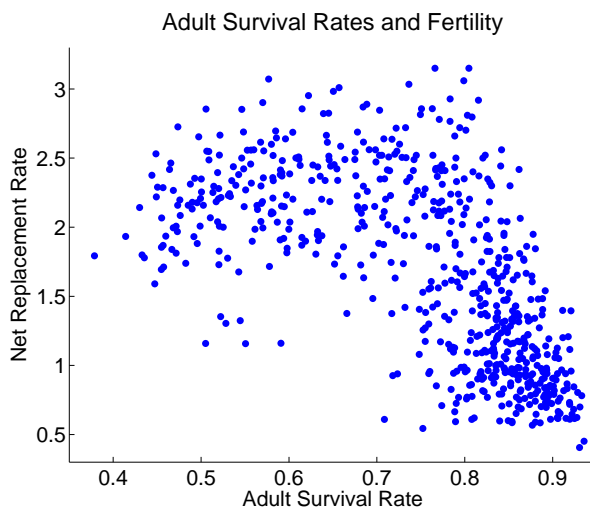
The strategy is to first estimate fertility as a linear function of the adult survival rate neglecting the interaction between life expectancy and primary education. Then we re-estimate the same equation but include an interaction term between primary schooling enrollment and adult survival rates. The estimated equations are of the form

$$\ln Nrr_{it} = \tilde{\beta}_0 + \tilde{\beta}_1 Sr_{it} + \tilde{\beta}_2 Edu_{it} + \tilde{\beta}_3 X_{it} + \epsilon_{it} + \gamma_i, \quad (37)$$

$$\ln Nrr_{it} = \beta_0 + \beta_1 Sr_{it} + \beta_2 Sr_{it} Edu_{it} + \beta_3 Edu_{it} + \beta_4 X_{it} + \epsilon_{it} + \gamma_i, \quad (38)$$

²⁷Estimating the net replacement rate as a quadratic function of adult survival rates leads in all specifications to a positive linear and negative quadratic coefficient indicating that there is a non-monotonic relationship.

Figure 3: Net Replacement Rate and Adult Survival Rate



Data sources: Net Replacement Rate: United Nations (2007). Adult Survival Rates: The World Bank (2010).

where Sr is the adult survival rate, Edu is a measure for education, X includes GDP per capita and a measure for secondary education. The error terms are denoted by ϵ and γ . For the first equation we expect that $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are negative. Including $SrEdu$ and estimating the second equation we expect that the sign of β_1 turns now positive and the sign of the interaction term β_2 negative and larger (in absolute values) than the coefficient on the survival rate. From the theory we derive that for zero education the correlation between survival rates and fertility is positive ($=\beta_1 > 0$) but for a sufficiently high primary schooling participation rate the total effect of rising survival rates is negative ($=\beta_1 + \beta_2 < 0$). However, including an interaction term between adult survival rates and secondary education should leave all signs unchanged (i.e. $\beta_1 < 0$, $\beta_3 < 0$). To control for country specific unobserved heterogeneity we run a panel regression with country fixed-effects but repeat the exercise also in a pooled OLS setting. To account for possible time trends in fertility we use a set of year dummies for all countries.²⁸

Table 2 presents the results from the models without interacting survival rates and primary education. As expected, higher survival rates and more education (primary and secondary) are negatively correlated with fertility. Including time trends, GDP per capita or secondary education and changing the estimation technique does not change the conclusion.

Table 3 contains essentially the same regressions but includes an interaction term between

²⁸See appendix for a robustness check with TFR as an alternative fertility measure, including other control variables (growth of GDP per capita, region specific fertility trends, etc.) and a detailed description of how the data set is constructed.

Table 2: Relationship Between Survival Rates and Fertility - No Interaction Effects

Survival Rate	-2.74***	-1.63***	-1.11***	-1.14***
Prim. Education	0.21*	0.21**	0.34***	-0.01
GDP/Capita	-13.35***	-4.27	-3.67	0.91
Years Sec. Schooling		-0.17***	-0.10***	-0.07***
Trend	–	–	yes	yes
Model	FE	FE	FE	OLS
Constant	3.38***	2.07***	1.28***	1.24***
N	469	469	469	465

Dependent variable is the log of crude birth rate. Primary education, years of secondary schooling, survival rate in levels, GDP/Capita in logs (coefficient \times 100). Trend is the set of year dummies. Stars indicate levels of significance (*: 10%, **: 5%, ***: 1%), robust standard errors. FE: country fixed effects. OLS: pooled cross section.

survival rates and education. In columns 1-4 we interact primary, and in columns 5-8 secondary education with adult survival rates. The signs of the coefficients are in line with our expectations. Higher survival rates are correlated with higher birth rates for low values of primary education. The relationship between years of secondary education and fertility is always negative. Further, the coefficient on the interaction term β_2 is larger (in absolute values) than the coefficient on the survival rates β_1 indicating that $\tilde{\mu}$ is in the support of the data: the effect of higher survival rates on fertility is initially positive but for full primary schooling negative. Repeating the same exercise but using years of secondary education instead of primary education does not change the sign of β_1 : countries with higher survival rates have lower fertility rates. Further, the interaction term is negative indicating that this relationship is stronger for higher life expectancy.

To sum up, the results point into the direction that the level of primary schooling matters for the sign of the correlation between parents' life expectancy and their fertility. Re-estimating the same model interacting secondary schooling with adults' survival rates does not change the sign of the survival rate. We interpret this as evidence in favor of the model presented in this paper but recognize that we fail to establish reliable causal evidence.

6 Conclusion and Discussion

This paper proposed a simple model arguing that to understand the change in agents's behavior during the demographic transition, it is crucial to account for changing nature of the costs of child quality. We show that if the input in children's human capital production is

Table 3: Relationship Between Survival Rates and Fertility - With Interaction Effects

Measure for Education →	Primary Schooling				Secondary Schooling			
Survival Rate	0.87	1.11***	2.52***	2.49***	-1.43***	-1.53***	-1.04***	-0.93***
Education	2.78***	2.24***	2.90***	2.60***	0.20	0.16	0.15	0.12
Surv. Rate × Educ.	-4.32***	-3.40***	-4.19***	-4.34***	-0.43**	-0.39**	-0.29	-0.23*
GDP/Capita	-7.15	-0.27	2.13	0.91	-0.59	-0.91	-1.00	1.26
Education _{-i}		-0.16***	-0.05*	-0.05***		0.11	0.27***	-0.08
Trend	–	–	yes	yes	–	–	yes	yes
Model	FE	FE	FE	OLS	FE	FE	FE	OLS
Constant	0.78	0.15	-1.49***	-0.89***	1.75***	1.76***	1.04**	1.11***
N	469	469	469	465	469	469	469	465

Dependent variable is the log of crude birth rate. When Education is the share of children with primary education, Education_{-i} denotes the average years of secondary schooling in the population (and vice versa). Education, Education_{-i}, survival rate and GDP/Capita in logs (coefficient × 100). Trend is the set of year dummies. Stars indicate levels of significance (*: 10%, **: 5%, ***: 1%), robust standard errors. FE: country fixed effects. OLS: pooled cross section.

only parental time, increasing life expectancy always increases fertility. This is because the price of child quality and quantity rise simultaneously with higher life expectancy.

A behavioral change can only occur if parents decide to educate their children in the formal system. This transforms time costs into monetary costs. Then, higher lifetime income “buys” also more time. In other words, if parents spend their own time to enhance children’s human capital, rising life expectancy increases the price of quality and quantity. With investment into child human capital via a school, increasing labor supply and adult human capital increases only the opportunity costs of quantity but leaves the price of quality unchanged. Hence, if parental human capital is sufficiently productive and the marginal valuation of an additional child is sufficiently low, the rising relative price of quantity will bias the parental decision towards more investment into quality and lower quantity.

Since at early stages of development, the share of people deciding to educate their children at home is high, gains in adult life expectancy initially increase fertility. As life expectancy rises, more agents decide to send their children to schools, thereby strengthening the composition effect but at the same time also reinforcing the negative effect of a higher life expectancy on fertility by a potential behavioral change. Once the share of parents participating the formal system is high enough, fertility will fall. We also find empirical evidence supporting the theory developed in this paper. Furthermore, we have proposed a theory why a formal schooling system may emerge endogenously without intervention by the state. We do not, however, make the next step and model why the society – via government and parliament

– decided set up a free public schooling system financed by taxes. The extension by such a political economy element is left for future research.

Appendix 1: Proofs

This appendix contains all analytical proofs. We compute the comparative statics by implicitly differentiating the system of first order conditions. Then, for a change in variable X , the partial derivatives of n and h are given by

$$\begin{bmatrix} h_X \\ n_X \end{bmatrix} = - \begin{bmatrix} F_{hh} & F_{hn} \\ F_{nh} & F_{nn} \end{bmatrix}^{-1} \begin{bmatrix} F_{hX} \\ F_{nX} \end{bmatrix} = -|A|^{-1} \begin{bmatrix} F_{nn}F_{hX} & -F_{hn}F_{nX} \\ -F_{nh}F_{hX} & F_{hh}F_{nX} \end{bmatrix} \quad (39)$$

Proof of proposition 1. From (17) we have that e^{if} is constant. Then we can write the household problem in terms of only n and h .

$$F_h = \frac{1}{c} \left(f(h) - \mu \bar{e}^\nu h^{\theta-1} \ell \right) \quad F_n = \frac{\beta}{n} (nz(e))^{1-\sigma} - \frac{T(e+\phi)}{\ell} \quad (40)$$

with $\ell \equiv T - h - n(e + \phi)$ and $c \equiv (\omega f(h)\ell)T^{-1}$. Partial derivatives are given by

$$\begin{bmatrix} h_T \\ n_T \end{bmatrix} = - \begin{bmatrix} -\frac{T}{h^2} \left(\theta + \frac{h^2}{\ell^2} \right) & -T \frac{e+\phi}{\ell^2} \\ -T \frac{e+\phi}{\ell^2} & -\frac{\sigma\beta}{n^2} (nz(e))^{1-\sigma} - \frac{T(e+\phi)^2}{\ell^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\theta}{h} + \frac{T-\ell}{\ell^2} \\ \frac{(e+\phi)(T-\ell)}{\ell^2} \end{bmatrix} \quad (41)$$

and the determinant $|A| = F_{hh}F_{nn} - F_{hn}F_{nh} > 0$ can be shown to be positive implying that we have a maximum. Combining the elements from above establishes $n_T > 0$ and $h_T > 0$. \square

Proof of proposition 2. The system of first order conditions is

$$F_h = \frac{1}{c} \left(f(h) - \mu \bar{e}^\nu h^{\theta-1} \ell \right) \quad F_n = \frac{\beta}{n} (nz(e))^{1-\sigma} - \frac{1}{c} (pe + \phi \omega f(h)) \quad (42)$$

$$F_e = n \left(\beta e^{\gamma-1} (nz(e))^{-\sigma} - \frac{p}{c} \right) \quad (43)$$

with $\ell \equiv T - h - \phi n$ and $c \equiv (\omega f(h)\ell - npe)T^{-1}$. Combining F_e and F_n one obtains $e = f(h) \frac{\phi}{p} \frac{\gamma}{1-\gamma}$. Substituting this into the FOCs reduces the dimension of the system by one equation. We proceed by using F_h and F_n . For the comparative statics we have

$$\begin{bmatrix} h_T \\ n_T \end{bmatrix} = - \begin{bmatrix} -\frac{1}{c} \left(\frac{1+\theta}{\theta} \omega \mu \bar{e}^\nu h^{\theta-1} \right) & -\frac{1}{c} \omega \mu \bar{e}^\nu h^{\theta-1} \phi \\ -\left(\frac{\omega f(h)}{c} \right)^2 \frac{\phi}{T(1-\gamma)} + (1-\sigma)\beta\gamma\theta \frac{(nz(e))^{1-\sigma}}{nh} & -\frac{\beta\sigma(nz(e))^{1-\sigma}}{n^2} + \left(\frac{\omega f(h)}{c} \frac{\phi}{1-\gamma} \right)^2 \frac{1}{T} \end{bmatrix}^{-1} \\ \times \begin{bmatrix} \frac{\omega \mu \bar{e}^\nu h^{\theta-1}}{c} \\ \left(\frac{\omega f(h)}{T(1-\gamma)c} \right)^2 \phi (h(1-\gamma) + \phi n) \end{bmatrix} \quad (44)$$

It can be again shown that $|A| = F_{hh}F_{nn} - F_{hn}F_{nh} > 0$. Further we have

$$h_T = |A|^{-1} \left[\beta \sigma \frac{(nz(e))^{1-\sigma}}{n^2} \left(\frac{\omega f(h)\phi}{(1-\gamma)cT} \right)^2 \omega f(h)(1+\gamma\theta) \right] > 0 \quad (45)$$

$$n_T = |A|^{-1} \left[\left(\frac{\phi \omega f(h)}{c(1-\gamma)T} \right)^2 \frac{n}{h} \omega f(h)(1+\gamma\theta) + \beta \gamma \theta^2 (1-\sigma) \frac{\omega f(h)(nz(e))^{1-\sigma}}{nc} \right] \geq 0 \quad (46)$$

where h_T is always positive and n_T is positive for $\sigma \leq 1$ but may be negative otherwise. \square

Proof of Proposition 3. Substituting optimal choices for e and n into the utility function, utility of agents conditional on their education system choice is

$$U^{if} = T \log \left(\omega \mu \bar{e}^\nu \frac{h^{1+\theta}}{T\theta^2} \right) + \frac{\beta (nz(e))^{1-\sigma}}{1-\sigma} \quad e = \frac{\phi \gamma}{1-\gamma} \quad (47)$$

$$U^{fo} = T \log \left(\omega \mu \bar{e}^\nu \frac{h^\theta (h + (h-T)\gamma\theta)}{(1-\gamma)T\theta^2} \right) + \frac{\beta (nz(e))^{1-\sigma}}{1-\sigma} \quad e = \frac{\omega \mu \bar{e}^\nu \phi \gamma h^\theta}{p \theta (1-\gamma)}. \quad (48)$$

and for a given vector $\{p, \omega\}$ the threshold ability level $\tilde{\mu}$ is implicitly defined by setting $U^{if} = U^{fo}$. Since relative sub-utility from consumption is not affected directly by ω , \bar{e} or μ (shift consumption proportionally), the decision which system to adopt depends only on μ *via investment into education* (income effect). Note that a higher (lower) price p requires a proportionally higher (lower) ability μ to restore indifference (the allocation does not change for $j = if$). Write the indifference condition as

$$T \log \left(\frac{c^{fo}}{c^{if}} \right) = \beta \left[u(n^{if} z(e^{if})) - u(n^{fo} z(e^{fo})) \right]. \quad (49)$$

Since the decision to join the formal system is based purely on a sufficiently large income effect, it holds that $c^{fo} \geq c^{if}$ for all solutions. For $T \rightarrow \infty$ we have

$$\begin{aligned} n^{if} &= \left(\frac{\beta(1-\gamma)z(e^{if})^{1-\sigma}}{(1+\theta)\phi} \right)^{\frac{1}{\sigma}} \\ h^{if} &= T \frac{\theta}{1+\theta} \end{aligned} \quad (50)$$

with e^{if} as defined above. For indifference it must always hold that $u(n^{if} z(e^{if})) - u(n^{fo} z(e^{fo})) \geq 0$. Given constant $u(\cdot)^{if}$ in the limit, we need that $u(\cdot)^{fo}$ is also constant with the difference approaching zero.²⁹ We also know that $\partial h^{fo}/\partial T > 0$ and hence $\partial e^{fo}/\partial T > 0$. Using $n^{if} > n^{fo}$

²⁹ e^{fo} and/or n^{fo} cannot grow monotonically otherwise the condition $u(\cdot)^{if} - u(\cdot)^{fo} \geq 0$ would be violated for some T .

and $e^{fo} > e^{if}$ we know that it must hold that n^{fo} approaches n^{if} from below and e^{fo} approaches e^{if} from above. For given p and ω , this can only happen if the threshold ability level $\tilde{\mu}$ is decreasing. Since utility is non-decreasing in μ , this must hold for all μ and T . \square

Proof of Lemma 1. Assume that we have found a vector $\{p, \omega, \tilde{\mu}\}$ such that $U^{if} = U^{fo}$ holds. Further, we can rewrite F_n for both schooling systems to

$$(1 - \gamma)^\sigma \frac{T\phi\theta}{\beta} = z(e)^{1-\sigma} \frac{h(1-\gamma)}{n(h)^\sigma} \quad \text{if } j = if, \quad (51)$$

$$\frac{T\phi\theta}{\beta} = z(e(h))^{1-\sigma} \frac{h + (h-T)\gamma\theta}{n(h)^\sigma} \quad \text{if } j = fo. \quad (52)$$

The LHS if $j = if$ is always smaller than the LHS if $j = fo$ and the same ordering must hold for the RHS in equilibrium. Since RHS is increasing in h , agents switching to the formal system have lower fertility and invest more in child human capital. \square

Proof of Lemma 2. Differentiating FOCs with respect to ability gives

$$F_{h\mu} = 0 \quad F_{n\mu} = 0 \quad \text{if } j = if \quad (53)$$

$$F_{h\mu} = 0 \quad F_{n\mu} = \frac{\beta\gamma(1-\sigma)}{n\mu} (nz(e))^{1-\sigma} \quad \text{if } j = fo \quad (54)$$

Combining this with the Hessian from above proves that ability does not change households' allocations. For $j = fo$ and assumption 2 more able agents invest more into adult human capital and lower fertility. Higher e follows from (21). \square

Proof of Proposition 4. First, the equilibrium price of education is given by equating supply and demand and solving for p which is then

$$p = \omega \left(\frac{\phi\epsilon\gamma}{1-\gamma} \frac{\int_{\tilde{\mu}}^1 f(h(\mu))n(\mu)d\mu}{\tilde{\mu} \left(\frac{m(f(h(T)))}{f(h(T))} \right)^{\frac{\epsilon}{1-\epsilon}} m(f(h(T))) + \int_{\tilde{\mu}}^1 \left(\frac{m(f(h(\mu,T)))}{f(h(\mu,T))} \right)^{\frac{\epsilon}{1-\epsilon}} m(f(h(\mu,T)))d\mu} \right)^{1-\epsilon} \quad (55)$$

Using the price of education from above to express the equilibrium investment into education by parents choosing the formal system gives

$$e = \phi \frac{\gamma}{1-\gamma} \frac{\omega f(h(\tilde{\mu}))}{p} \quad (56)$$

Substituting this into (49) gives

$$T \log \left(\frac{c^{f^o}}{c^{if}} \right) = \beta \frac{z(e^{if})^{1-\sigma}}{1-\sigma} \left[(n^{if})^{1-\sigma} - (n^{f^o} z(\Phi(\tilde{\mu}, T))^{f^o})^{1-\sigma} \right] \quad (57)$$

$$\Phi(\tilde{\mu}, T) \equiv f(h(\tilde{\mu}, T)) \times \quad (58)$$

$$\left(\frac{1 - \gamma \tilde{\mu} \left(\frac{m(f(h(T)))}{f(h(T))} \right)^{\frac{\epsilon}{1-\epsilon}} m(f(h(T))) + \int_{\tilde{\mu}}^1 \left(\frac{m(f(h(\mu, T)))}{f(h(a, T))} \right)^{\frac{\epsilon}{1-\epsilon}} m(f(h(a, T))) da}{\phi \epsilon \gamma \int_{\tilde{\mu}}^1 f(h(\mu)) n(\mu) d\mu} \right)^{1-\epsilon} \quad (59)$$

We know that the LHS is positive for all T . Since n^{if} is converging to a constant, the same must hold for $u(n^{f^o} z(e^{f^o}))$. With n^{f^o} approaching n^{if} from below, $\Phi(\tilde{\mu}, T)$ must converge to a constant from above. Then, the dynamics of the threshold ability level depends on $\partial \tilde{\mu} / \partial T = -(\partial \Phi / \partial T) / (\partial \Phi / \partial \tilde{\mu})$. We know $\partial \Phi / \partial T < 0$ and it can be shown that the sign of $\partial \Phi / \partial \tilde{\mu}$ depends on the sign of

$$\left(\frac{m(f(h(T)))}{f(h(T))} \right)^{\frac{\epsilon}{1-\epsilon}} m(f(h(T))) - \left(\frac{m(f(h(\tilde{\mu}, T)))}{f(h(\tilde{\mu}, T))} \right)^{\frac{\epsilon}{1-\epsilon}} m(f(h(\tilde{\mu}, T))). \quad (60)$$

If this is positive we have $\partial \Phi / \partial \tilde{\mu} > 0$, otherwise the sign is indeterminate. With $m(\cdot) = 1$, we have $f(h(T))^{-\frac{\epsilon}{1-\epsilon}} - f(h(\tilde{\mu}, T))^{-\frac{\epsilon}{1-\epsilon}} > 0$ (because $f(h(T)) < f(h(\tilde{\mu}, T))$) implying that the ability level is increasing. By monotonicity of the RHS in $\tilde{\mu}$ this holds for all T . For m being a linear function, i.e. $m(\cdot) = f(\cdot)$ we have that $f(h(T)) - f(h(\tilde{\mu}, T)) < 0$ allowing for the possibility of $\partial \Phi / \partial \tilde{\mu} < 0$ and hence a falling ability threshold. \square

Proof of Corollary 1. If fertility is a concave function of T , there are two life expectancies T_l and T_h at which fertility per family equals 2 and there must be an intermediate life expectancy T_{im} which maximizes fertility (above 2 children per couple) and population growth rate. Concavity gives us $\frac{\partial n(T_l)}{\partial T_l} > 0$ and $\frac{\partial^2 n(T_h)}{\partial T_h^2} < 0$ resulting in rising and falling population growth rate at the two stationary population levels ($g_P = 0$).

$$\left. \frac{\partial g_P}{\partial T_t} \right|_{g_P=0} = \frac{\partial n(T_t) / \partial T_t}{1 + T_t} \quad (61)$$

Maximum population growth is defined by (62) and the derivative of g_P at that point is given by (63). Thus, g_P increases at T_l , attains a maximum at T_{im} and starts to decrease thereafter

and becomes even negative at T_h .

$$1 - (1 + g_P)^{T_{im}} + T_{im} \log(1 + g_P)^{T_{im}} = 0 \quad (62)$$

$$\left. \frac{\partial g_P}{\partial T_{im}} \right|_{g_P > 0} = -\frac{(1 + g_P) \log(1 + g_P)}{T_{im}} < 0 \quad (63)$$

□

Extensions

We have assumed that agents' labor productivity does not affect their productivity as parents (teachers). In this section we relax this simplifying assumption. For the sake of brevity, we do this only for the case $j = if$ but a similar reasoning applies to the case $j = fo$. We show that allowing for a correlation between parental ability and teaching productivity has implications only for the cross-sectional distribution but does not affect the qualitative results of the paper (i.e. the reaction of fertility if adult life expectancy increases).

To allow for a positive effect of labor market productivity on “teaching ability” we consider two polar modeling environments. In the first case, we assume that e units of time spent yield e/μ effective units of time. Thus, more productive parents need less time to teach the same amount of “effective hours”. The other alternative is to introduce parental productivity directly into the human capital production function z and leave the budget constraint unchanged.

To model the first scenario, we write the budget constraint for $j = if$ as

$$Tc^j \leq \omega f(h^j) (T - (e^j/\mu + \phi) n^j - h^j) \quad \text{if } j = if. \quad (64)$$

Then we can derive that

Lemma 3. *If children are educated in the informal system, higher ability increases adult and child human capital and decreases fertility. If adult life expectancy increases, adult schooling and fertility will increase. Investment into child human capital is constant.*

Proof. See below. □

Now we move on to the second alternative. Keeping the budget constraint unchanged, we allow for a more general relationship between investment into children and their human

capital and write the production function as $z(e, \mu)$ with positive partial and cross derivatives. Equilibrium child human capital investment for $j = if$ is implicitly defined by

$$e + \phi - \frac{z(e, \mu)}{z_e(e, \mu)} = 0, \quad (65)$$

and we show in the proof below that if the elasticity of z is increasing in parental ability, investment into children's human capital is higher for smarter parents. Then we can state that

Lemma 4. *If children are educated in the informal system, higher ability increases investment into child human capital. The effect of ability on adult human capital and fertility is ambiguous. If adult life expectancy increases, adult schooling and fertility will increase. Investment into child human capital is constant.*

Proof. See below. □

Thus, introducing a positive correlation between labor market and teaching ability does not change the comparative statics with respect to changes in adult life expectancy but has only cross-sectional implications. Rising life expectancy will still increase fertility and human capital investment.

Proof of Lemma 3. Investment into children's human capital is $\hat{e} \equiv \mu\phi \frac{\gamma}{1-\gamma} = \mu e$ implying that more able parents invest more into their offspring's human capital. The remaining FOCs are

$$F_h = \frac{1}{c} \left(f(h) - \mu h^{\theta-1} \ell \right) \quad F_n = \frac{\beta}{n} (nz(\hat{e}))^{1-\sigma} - \frac{T(e + \phi)}{\ell} \quad (66)$$

Further, we know that the sign of the elements in the Hessian from (41) do not depend on μ (which proves the second sentence). Given assumption 2 it can be shown that $F_{n\mu} < 0$ and $F_{h\mu} = 0$. Then we have

$$\begin{bmatrix} h_\mu \\ n_\mu \end{bmatrix} = -|A|^{-1} \begin{bmatrix} -F_{hn}F_{n\mu} \\ F_{hh}F_{n\mu} \end{bmatrix} \begin{matrix} > 0 \\ < 0 \end{matrix} \quad (67)$$

□

Proof of Lemma 4. Parents' ability changes optimal investment into child human capital via

$$e_\mu = -\frac{e \frac{\partial \epsilon(\mu)}{\partial \mu}}{1 - \frac{1}{\epsilon(\mu)}} \geq 0 \quad (68)$$

where $\epsilon(\mu)$ is the elasticity of child human capital production function with respect to e . For the sake of clarity we assume that it depends only on μ and not on e . Given that for a solution we need $\epsilon(\mu) < 1$, e rises if the elasticity of z is increasing in μ . Differentiating the new optimality conditions w.r.t. ability gives

$$F_{h\mu} = -\frac{ne_\mu T}{\ell^2} \quad F_{n\mu} = e_\mu \left(\beta(1-\sigma)(nz(e))^{-\sigma} z' - T \frac{(T-h)}{\ell^2} \right) + \beta(1-\sigma)(nz(e))^{-\sigma} z_\mu. \quad (69)$$

Using the Hessian from (41) establishes that

$$\begin{bmatrix} h_\mu \\ n_\mu \end{bmatrix} = -|A|^{-1} \begin{bmatrix} T \frac{e+\phi}{\ell^2} \beta(1-\sigma)(nz(e))^{-\sigma} z_\mu \\ T e_\mu \left(\theta \left(\frac{T}{h\ell} \right)^2 - \left(\frac{\theta}{h^2} + \frac{1}{\ell^2} \right) \beta(nz(e))^{-\sigma} (1-\sigma) z' \right) - T \left(\frac{\theta}{h^2} + \frac{1}{\ell^2} \right) \beta(nz(e))^{-\sigma} (1-\sigma) z_\mu \end{bmatrix} \begin{matrix} \geq 0 \\ \geq 0 \end{matrix} \quad (70)$$

where the sign of the derivatives depend on on σ , e_μ and z_μ . □

Appendix 2: Empirical Evidence

This appendix provides a robustness check using different fertility measures and additional control variables. Further, we explain the construction of our variables.

Schooling

Our data for education are taken from Barro and Lee (2010). To construct a measure for the primary schooling attainment rate, we use the share of agents without any education (coded *lu*). Since there is no information about the primary school attendance rate or children of school entry age in year t , we use the share of agents aged 15-19 in ten years to proxy for the primary schooling enrollment rates of children today. As a measure of adult human capital, we use the years of secondary schooling of agents older than 15 (coded *yr_sch_sec*).

Adult Life Expectancy

Firstly, since we only need one aggregate measure for life expectancy, we do not use gender specific survival rates but compute an average from data for men and women. The weights are given by the respective population shares in that period. Secondly, since the survival rate is given for a 15 year old agent for a given year t but fertility is an aggregate measure comprising different cohorts, we have to adjust one of the two variables in order to synchronize the timing of the model and the data. We adjust the survival used in the empirical analysis such that the survival rate in year t is given by

$$SR_t = \sum_{j=0}^J \omega_j SR_{t-j} \quad (71)$$

where j denotes an age-group, t is calendar time, and ω is a weight for each age-group. We compute the weights from life-cycle fertility rates from the United Nations (2007). As an example, assume that 50% of the women have their children between the ages 15-25 and 50% between 25-35. In such a case, fertility in year t is a function of the survival rate in year t and the survival rate in year $t - 10$. This is because the fertility of currently “young” women (aged 15-25) is determined by their survival rate in year t but the fertility of the women aged 25-35 is given by their life expectancy which is given by SR_{t-10} since they make their fertility decision at each age during their life-cycle as a function of their cohort-specific life expectancy.

Fertility Measure and Additional Variables

To test the robustness of our results, we added the growth rate of GDP/capita as an additional regressor and to control for region-specific fertility trends we include a separate fertility trend for “Western” countries (Europe, US, Canada, Australia, New Zealand and Japan) all other countries.

Tables 4 and 5 show the results with NRR as the fertility measure but now also including the growth rate of GDP per Capita as an additional regressor. In tables 6 and 7 we display the results with the Total Fertility Rate (TFR) as the alternative fertility measure. As can be seen, the qualitative results are identical to the ones presented in the paper.

Table 4: Relationship Between Survival Rates and NRR - No Interaction Effects

Survival Rate	-2.47***	-1.74***	-1.23***	-1.20***
Prim. Education	0.18*	0.19**	0.32***	0.06
GDP/Capita	-17.66***	-5.48	-5.48	0.00
Growth GDP/Capita	1.37***	0.30	0.32	-1.23**
Years Sec. Schooling		-0.15***	-0.08**	-0.06***
Trend	–	–	yes	yes
Model.	FE	FE	FE	OLS
Constant	3.55***	2.23***	1.50***	1.31***
N	432	432	432	428

Table 5: Relationship Between Survival Rates and NRR - With Interaction Effects

Measure for Education →	Primary Schooling				Secondary Schooling			
Survival Rate	0.82	.96**	2.23***	2.12***	-1.61***	-1.69***	-1.20***	-0.99***
Education	2.51***	2.16***	2.72***	2.46***	0.20	0.17	0.10	0.12
Surv. Rate × Educ.	-3.97***	-3.35***	-3.99***	-3.99***	-0.41**	-.04*	-0.21	-0.23
GDP/Capita	-11.50**	-1.55	1.12	0.21	-1.32	-1.60	-3.01	0.29
Growth GDP/Capita	1.22***	0.29	0.10	-1.24***	0.24	0.24	0.26	-1.22**
Education _{<i>i</i>}		-0.13***	-0.04	-0.05***		0.08	0.26**	-0.01
Trend	–	–	yes	yes	–	–	yes	yes
Model	FE	FE	FE	OLS	FE	FE	FE	OLS
Constant	1.18**	0.36	-1.19**	-0.68**	1.90***	1.91***	1.3***	1.18***
N	432	432	432	428	432	432	432	428

Table 6: Relationship Between Survival Rates and TFR - No Interaction Effects

Survival Rate	-2.95***	-2.10***	-1.48***	-1.64***
Prim. Education	0.04	0.05	0.21*	-0.10
GDP/Capita	-21.85***	-7.78	-6.74	-1.95
Growth GDP/Capita	1.48***	0.25	0.20	-1.32**
Years Sec. Schooling		-0.17***	-0.09**	-0.07***
Trend	–	–	yes	yes
Model.	FE	FE	FE	OLS
Constant	5.27***	3.74***	2.79***	3.08***
N	434	434	434	430

Table 7: Relationship Between Survival Rates and TFR - With Interaction Effects

Measure for Education →	Primary Schooling				Secondary Schooling			
Survival Rate	0.22	0.39	1.91***	1.85***	-2.11***	-2.00***	-1.43***	-1.41***
Education	2.29***	1.87***	2.57***	2.38***	0.25	0.29*	0.27	0.14
Surv. Rate × Educ.	-3.83***	-3.09***	-3.92***	-4.13***	-0.49***	-0.53***	-0.42**	-0.25*
GDP/Capita	-15.93***	-4.17	-0.26	-1.75	-2.45	-2.14	-1.72	-1.63
Growth GDP/Capita	1.34***	0.24	-0.02	-1.33***	0.16	0.16	0.07	-1.30**
Education _{-i}		-0.16***	-0.05	-0.05***		-0.10	0.09	-0.18**
Trend	–	–	yes	yes	–	–	yes	yes
Model	FE	FE	FE	OLS	FE	FE	FE	OLS
Constant	2.99***	2.02***	0.15	1.02***	3.29***	3.28***	2.39***	2.95***
N	434	434	434	430	434	434	434	430

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